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HYDRODYNAMIC COEFFICIENTS FOR VERTICAL CIRCULAR CYLINDERS AT FI--ETC(U)

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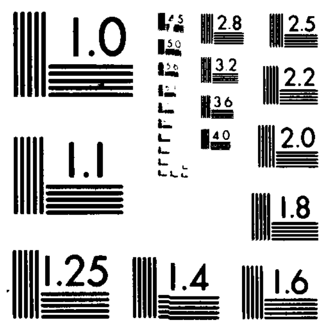

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United States Naval Academy
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HYDRODYNAMIC COEFFICIENTS FOR VERTICAL ~~FOR~~ CIRCULAR CYLINDERS AT FINITE DEPTH

by

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14) USNA-EW-11-80

Report No. EW-11-80

Approved for public release
Distribution Unlimited

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NOMENCLATURE

a	radius of the circular cylinder
a_{ij}	added mass or moment coefficient
d	gap between the cylinder and the bottom of the sea
D	volume of the cylinder
g	gravitational acceleration
h	depth of water
H_k	$J_k + i\ Y_k$ Hankel function
i	$\sqrt{-1}$
I_k	Modified Bessel function, first kind of order k
J_k	Bessel function first kind of order k
K_n	Modified Bessel function, second kind of order k
m_o, m_q	as defined by theequation 2.22, 2.23
$\vec{n}$	normal vector
$\vec{r}$	position vector
t	draft of the cylinder
v	velocity
x,y,z	cartesian coordinate system
Y_k	Bessel function, second kind of order k
Z_k	function defined by equation 2.18, 2.19
δ_{ij}	Kronecker delta
Θ	angular position in cylindrical coordinates
ρ	fluid density
$\bar{\Phi}, \Phi$	velocity potentials
ν	$\omega^2 a/g$
ω	radian frequency

HYDRODYNAMIC COEFFICIENTS FOR VERTICAL CIRCULAR CYLINDERS AT FINITE DEPTH

ABSTRACT

Hydrodynamic coefficients for vertical circular cylinders at finite water depth are obtained and presented for different depth to radius and draft to radius ratios. A summary of equations for computer application is also presented. Limiting values for heave added mass for zero frequency is also discussed.

1. INTRODUCTION

Vertical circular cylinders are used in many oceanographic applications such as buoys, drilling rigs and instrumentation platform for their simplicity in construction. The available data on their hydrodynamic coefficients is limited and the present numerical procedures are based on finite element solutions or the numerical solutions of integral equations. The present method is relatively simple to formulate and the solution requires a very short computer time. The hydrodynamic coefficients such as added mass, damping coefficients for heave, sway and pitch motions are formulated and the results for different depth to radius and draft to radius values are presented in graphical form.

Havelock (1955) theoretically determined the added mass and damping coefficients for a sphere. Kim (1965) studied the hydrodynamic coefficients for elipsoidal bodies oscillating at the free surface. Shen Wang (1966) calculated the added mass and damping coefficients of sphere in infinite and finite depth of water. Garrison (1975) gave the general formulation of these coefficients for arbitrary forms in terms of distributed singularities and the numerical results for a vertical circular cylinder in infinite and finite depth of water. Bai and Yeung (1974) calculated added mass coefficients for horizontal and vertical cylinders. Bai (1976) gave the added mass and damping coefficients for axisymmetric ocean platforms Kritis (1979) had applied the hybrid integral method of Yeung to axisymmetric bodies and gave numerical results for a circular cylinder.

The various methods developed for the solution of three dimensional axisymmetrical bodies can be summarized as follows. In the first group of methods Sources and Multipoles are distributed inside the body and their strength is calculated to satisfy impermeable boundary conditions of the body. The second set of solutions distributes the singularities at the surface of the body and an integral equation is used through the use of Green's theorem. The solution of the integral equation gives the strength of the singularities. Thirdly, the finite element formulation is used to find the velocity potential at specified node points. Possible combinations such as the Hybrid method referred above also exist, combining the above solutions and reducing the computational effort. The present formulation follows the general procedure outlined by Garrett (1970) who studied the scattering of waves at the presence of circular docks.

Although it is of major concern to Naval Architects and ocean engineers very few data exist on the hydrodynamic coefficients of circular cylinders. Serving to this aim graphical results covering a large range of parameters and summary formulas for computer applications are presented in this paper.

2. FORMULATION AND SOLUTION OF THE PROBLEM

The coordinate system Oxyz is shown in Figure 1. The origin is at the bottom and z is positive upwards. The region $0 \le z \le h$ is assumed to be filled by an incompressible fluid of density ρ. The undisturbed free surface is at $z = h$. The radius of the cylinder is a and draft $T = h - d$, where d is the gap between the cylinder and the bottom. The standard small motion assumptions are made and the motion is periodic with frequency ω. An irrotational flow is assumed to exist given by

$$\tilde{\Phi}(r,\Theta,z;t) = \mathrm{Re}\{\phi(r,\Theta,z)\, e^{-i\omega t}\} \tag{2.1}$$

where r,Θ,z are cylindrical coordinates and $\Theta = o$ corresponds to the positive x axis. $\phi(r,\Theta,z)$ is a complex spatial velocity potential which satisfies:

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \Theta^2} + \frac{\partial^2}{\partial z^2}\right)\phi = o \tag{2.2}$$

in the fluid region.

The boundary conditions for different motions of the cylinder are given below.

At the free surface

$$\Phi_z - \frac{\omega^2}{h}\Phi = o \quad \text{at } z = h \tag{2.3}$$

$$\Phi_z = o \text{ at } z = o \tag{2.4}$$

$$\Phi_n = \vec{v}\cdot\vec{n} + \vec{\Omega}\cdot(\vec{r} \times \vec{n}) \tag{2.5}$$

on the body surface where $\vec{n}$ is the normal to the surface. For heave motion

$$\frac{\partial\Phi}{\partial z} = V_H \text{ on } z = d \tag{2.6}$$

and

$$\frac{\partial\phi}{\partial r} = o \text{ on } r = a \text{ for } d \leq z \leq h \tag{2.7}$$

where V_H is the velocity of the cylinder due to heaving motion alone.

For sway motion

$$\frac{\partial\Phi}{\partial z} = o \text{ on } z = d \tag{2.8}$$

$$\frac{\partial\Phi}{\partial r} = V_S \cos\Theta \quad \text{at } r = a \text{ for } d \leq z \leq h \tag{2.9}$$

where V_S is the velocity of the cylinder due to sway motion alone.

For pitching motion

$$\frac{\partial\Phi}{\partial z} = \Omega\, r \cos\Theta \tag{2.10}$$

$$\frac{\partial\Phi}{\partial r} = \Omega\,(h-z) \cos\Theta \tag{2.11}$$

where Ω is the angular speed of pitching motion.

Following Gerrett's method the fluid is divided into two parts namely the interior part ABCD and the exterior part ACEF as in Figure 1. The appropriate solution of velocity potential in each region is found and the solutions are matched at the boundary so as to have the continuity in ϕ and its first derivatives are satisfied. The velocity potential in the interior domain is expressed as

$$\phi = D^k [\phi_p^k (r,z) + \phi_h^k (r,z)] \cos K\Theta \quad \text{for } o<r<a \quad o<z<d \tag{2.12}$$

where k = o refers to heave motion and k = 1 refers to sway and pitch motion. ϕ_p^k and ϕ_h^k are the particular and homogeneous solutions in the interior region.

The particular solutions for different motions are given as

$$\phi_p^o (r,2) = \frac{1}{2d^2} (z^2 - \frac{r^2}{2}) \quad \text{for heave} \quad (*) \tag{2.13}$$

$$\phi_p^1 (r,2) = o \quad \text{for sway} \tag{2.14}$$

$$\phi_p^1 (r,2) = \frac{1}{2d^3} (r\, z^2 - \frac{r^3}{3}) \text{for pitch} \tag{2.15}$$

The particular solutions satisfy the respective kinematic conditions at the bottom of the cylinder and the bottom of the fluid for $r \leq a$ given by the equation (2.4) through (2.11). D^k is a constant of dimension $[L^2/T]$ and is chosen to fit the particular motion of the cylinder.

In the exterior region the velocity potential is given in terms of eigen expansion.

(*) A particular solution for heave motion is suggested by Professor J.N. Newman of M.I.T.

$$\phi = D^k \phi_e (r,z) \tag{2.16}$$

$$\phi_e = B_o^k H_k (m_o r) Z_o (z) + \sum_{q=1}^{\infty} B_q^k K_k (m_q r) Z_q (z) \tag{2.17}$$

where H_k is the Hankel function of the first kind of order K = 0,1 and K_k is the modified Bessel function of the second kind of order K = 0,1. B_o^k and B_q^k are complex unknowns. The orthonormal Z_i functions in the interval $o \leq z \leq h$ are defined as

$$Z_o (z) = N_o^{1/2} \cosh (m_o z) \tag{2.18}$$

$$Z_q (z) = N_q^{1/2} \cos (m_q z) \tag{2.19}$$

with

$$N_o = \frac{1}{2} [1 + \frac{\sinh (2 m_o h)}{2 m_o h}] \tag{2.20}$$

$$N_q = \frac{1}{2} [1 + \frac{\sin (2 m_q h)}{2 m_q h}] \tag{2.21}$$

where m_o and m_q are the solution of the equations

$$m_o \tanh (m_o h) = \frac{\omega^2}{g} \tag{2.22}$$

$$m_q \tan (m_q h) = - \frac{\omega^2}{g} \tag{2.23}$$

To satisfy the continuity of velocity potential at r = a for $o<z<d$ homogeneous solution defined above is used as follows

$$\phi_p^k (a,z) + \phi_h^k (a,z) = \phi_e^k (a,z) \tag{2.24}$$

$$\frac{\partial}{\partial r} [\phi_p^k (a,z) + \phi_h^k (a,z)] = \phi_e^k (a,z) \qquad (2.25$$

for $o<z<d$.

The value of homogeneous potential at r = a can be expanded as

$$\phi^k_h (a,z) = \frac{A_o^k}{2} + A_n^k \cos (\frac{n\pi z}{d}) \tag{2.26}$$

For continuity of potential function coefficients A_i^k's are obtained using (2.24) as:

$$A_n^k = \frac{2}{d} \int_o^d [\phi_e^k (a,z) - \phi_p^k (a,z)] \cos (\frac{n\pi z}{d}) dz \tag{2.27}$$

The particular solution is a known function therefore A_n^k can be expressed as

$$A_n^k = \frac{2}{d} \int_o^d \phi_e^k (a,z) \cos \left(\frac{n\pi z}{d}\right) dz - \alpha_n^k \tag{2.28}$$

where

$$\alpha_n^k = \frac{2}{d} \int_o^d \phi_p^k (a,z) \cos (n\pi z/d)\, dz \qquad k = 0.,$$

The interior homogeneous solution can be written now as

$$\phi_h^k (r,z) = \frac{A_o^k}{2} \left(\frac{r}{a}\right)^k + \sum_{n=1}^{\infty} A_n^k \frac{I_k(n\pi r/h)}{I_k\,(n\pi a/h)} \cos \left(\frac{n\pi z}{h}\right) \tag{2.30}$$

in $r \leq a$, $o \leq z \leq d$

Multiplying the equation (2.25) by Z_q (z) and considering that $\frac{\partial\phi}{\partial r}$ $(a,\theta,z) = V \cos\theta$ for $d \leq z \leq h$ and integrating between limits in o and h one obtains

$$B_o^k \cdot (h\, m_o)\, H_k'\, (m_o a) = \int_o^d \frac{\partial\, \phi_h^k}{\partial r} (a,z) \cdot Z_o\, (z)\, dz + \beta_o^k \tag{2.31-a}$$

$$B_q^k\, (m_q h) \cdot K_k\, (m_q a) \int_o^d \frac{\partial}{\partial r}{}_h^k\, (a,z) \cdot Z_q\, (z)\, dz + \beta_q^k \tag{2.31-b}$$

for $q = 1,2,...$

where

$$\beta_o^k = \int_o^d \frac{\partial\phi_p^k\, (a,z)}{\partial r} Z_o\, (z)\, dz + \frac{1}{DK} \int_d^h V\, Z_o\, (z)\, dz \tag{2.32}$$

$$\beta_q^k = \int_o^d \frac{\partial\phi_p^k(a,z)}{\partial r} Z_q\, (z)\, dz + \frac{1}{DK} \int_d^h V\, Z_q\, (z)\, dz \tag{2.33}$$

Where $V = 0$ for heave, $V = V_s$ for sway and $V = \Omega\,(h-z)$ for pitch motion.

Inserting the values of ϕ_e^k from (2.17) into equation (2.27) and ϕ_h^k from (2.28) into (2.31-a) and (2.31-b) we obtain

$$A_n^k = \{ B_o^k \cdot H_k\ (m_o a) \cdot E_{on} + \sum_{q=1}^{\infty} B_q^k \cdot K_k\ (m_q a)\ E_{qn} \} - \alpha_n^k \qquad (2.34)$$

for n = 0,1,2,...

$$B_o^k = \frac{\{ \frac{K\,d}{4a} A_o^k \cdot E_{oo} + \sum_{n=1}^{\infty} A_n^k \frac{n\pi}{2} \frac{I_k'(n\pi a/d)}{I_k(n\pi a/d)} \cdot E_{on} \}}{(m_o h)\ H_k'\ (m_o a)} + \frac{\beta_o^k}{(m_o h)\ H_k'\ (m_o a)} \qquad (2.35\text{-a})$$

$$B_q^k = \frac{\{ \frac{kd}{4a}\ E_{qo} \cdot A_o^k + \sum_{n=1}^{\infty} A_n^k \frac{n\pi}{2} \frac{I_k'(n\pi a/d)}{I_k(n\pi a/d)}\ E_{qn} \}}{(m_q h)\ K_k'\ (m_q a)} + \frac{\beta_q^k}{(m_q h)\ K_k'(m_q a)} \qquad (2.35\text{-b})$$

The equations (2.34), and (2.35-a,b) form a coupled system of equations where A_n^k and B_q^k are the unknowns. Substituting B_q from (2.35-a,b) into (2.34), a set of linear equations for A_n are obtained.

Thus:

$$\sum_{j=o}^{\infty} \gamma_{nj}^k\ A_n^k = -\ h_n^k \quad \text{for } n = 0,\ 1,\ 2,\ \ldots \qquad (2.36)$$

where

$$\gamma_{nj}^k = \{ [\frac{H_k(m_o a)}{H_k'(m_o a)} \cdot \frac{E_{on} \cdot E_{oj}}{m_o h} + \sum_{q=1}^{\infty} \frac{K_k(m_j a)}{K_k'(m_q a)} \cdot \frac{E_{qm} \cdot E_{qj}}{m_q h}]\ U_j^k - \delta_{nj} \} \qquad (2.37)$$

$$h_n^k = (\frac{H_k(m_o a)}{H_k'(m_o a)} \cdot \frac{\beta_o^k \cdot E_{on}}{m_o h} + \sum_{q=1}^{\infty} \frac{\beta_q^k \cdot E_{an}}{m_q h}) - \alpha_n^k \qquad (2.38)$$

where δ_{nj} kronecker delta ,

$$U_j^k = \frac{j\pi}{2}\ I_k'\ (j\pi a/d)/I_k(j\pi a/d), \qquad \text{for } j = 1,2,3,\ldots \qquad (2.39$$

$$U_o^k = \frac{kd}{4a}\ , \qquad (2.40$$

$$E_{qn} = \frac{2}{d} \int_o^d Z_q(z) \cos\ (n\pi z/d)\ dz\ . \qquad (2.41)$$

3. HYDRODYNAMIC FORCES AND MOMENTS IN TERMS OF THE VELOCITY POTENTIAL.

The forces and moments are defined by the integrals taken over the body surface as follows:

$$\vec{F} = -\rho \frac{\partial}{\partial t} \iint_S \bar{\dot{\Phi}} \cdot \vec{n} \, ds \quad (3.1)$$

$$\vec{M} = -\rho \frac{\partial}{\partial t} \iint \bar{\Phi} \cdot (\vec{r} \times \vec{n}) \, ds \quad (3.2)$$

The added mass and damping coefficients are calculated in the following manner.

$$\frac{a_{11}}{\rho D} + i \frac{b_{11}}{\omega\rho D} = \frac{D^1}{\rho D} \iint_S \Phi^1(r,\Theta,z) \, n_1 \, ds \quad (3.3)$$

$$\frac{a_{16}}{\rho D} + i \frac{b_{16}}{\omega\rho D} = \frac{D^1}{\rho D} \iint_S \Phi^1 (r,\Theta,z) \cdot (\vec{r} \, \vec{n}) \, ds \quad (3.4)$$

$$\frac{a_{22}}{\rho D} + i \frac{b_{22}}{\omega\rho D} = \frac{D^o}{\rho D} \iint_S \Phi^o(r,\Theta,z) \, n_2 \, ds \quad (3.5)$$

$$\frac{a_{66}}{\rho D} + i \frac{b_{66}}{\omega\rho D} = \frac{D^1}{\rho D} \iint_S \Phi^1 (r,\Theta,z) \, (\vec{r} \times \vec{n}) \, ds \quad (3.6)$$

$$\frac{a_{61}}{\rho D} + i \frac{b_{61}}{\omega\rho D} = \frac{D^1}{\rho D} \iint_S \Phi^1 (r,\Theta,z) \, n_1 \, ds \quad (3.7)$$

where D is the volume of the cylinder, a_{11} and b_{11} are sway added mass and damping coefficients and a_{16} and b_{16} are sway induced pitch added moment and damping coefficient, a_{22} and b_{22} are the heave added mass and damping coefficients, a_{61} and b_{61} pitch induced added mass and damping coefficients. Here Φ^1 in (3.3) and (3.4) is the potential for sway motions, and Φ^1 in (3.6) and (3.7) is the potential for pitch motion.

For reasons of symmetry

$$a_{16} = a_{61}$$

$$b_{16} = b_{61}$$

$$a_{44} = a_{66}$$

$$b_{44} = b_{66}$$

The heave added mass and damping coefficients in particular are given by

$$\frac{a_{22}}{\rho D} + i\,\frac{b_{22}}{\rho D} = \frac{2\pi d}{\rho D}\int_0^a [\,\phi_p^o\,(r,d) + \phi_h^o\,(r,d)]\,dr \qquad (3.8)$$

Using ϕ_p^o and ϕ_h^o form quation (2.13) and (2.30) into (3.8) and integrating

$$\frac{a_{22}}{\rho D} + i\,\frac{b_{22}}{\rho D} = \frac{d}{h-d}\left\{\frac{1}{2} - \frac{1}{8}\left(\frac{a}{d}\right)^2 + \frac{1}{2}A_o^o + \frac{2}{\pi}\left(\frac{d}{a}\right)\sum_{n=1}^{\infty}\frac{A_n^o(-1)^n}{n}\,\frac{I_1(n\pi a/d)}{I_o(n\pi a/d)}\right\} \qquad (3.9)$$

Similar calculations are made for other added mass and damping coefficients and these formulas are given below.

$$\frac{a_{11}}{\rho D} + i\,\frac{b_{11}}{\rho D} = \left\{B_o^1\,\frac{H_1(m_oa)\,[sh(m_oh)-shm_od]}{N_o^{1/2}\,m_o(h-d)} + \sum_{q=1}^{\infty} B_q^1\,\frac{K_1(m_qa)[\sin(m_qh)-\sin(m_qd)]}{N_q^{1/2}\,m_q(h-d)}\right. \qquad (3.10)$$

$$\frac{a_{66}}{\rho D} + i\,\frac{b_{66}}{\rho D} = \frac{d^3}{(h-d)a^3}\left\{\frac{d}{a}\left[t_o\cdot N^{-1/2}H_1(m_oa)\cdot B_o^1 + \sum_{q=1}^{\infty} t_q\,N_q^{-1/2}\,K_1\,(m_qa)\,B_q^1\right] + \right.$$
$$-\left[\frac{1}{8}\left(\frac{a}{d}\right)^2\left(1-\frac{1}{6}\left(\frac{2}{d}\right)^2\right) + \frac{1}{8}\left(\frac{a}{d}\right)A_o^1 + \frac{1}{\pi}\sum_{n=o}^{\infty}\frac{(-1)^n}{n}A_n\,\frac{I_2\,(n\pi a/d)}{I_1\,(n\pi a/d)}\right]\Big\}$$

$$\frac{a_{16}}{\rho Da} + i\,\frac{b_{16}}{\omega\rho Da} = \frac{d}{h-d}\left\{\left[t_oN_o^{-1/2}\,H_1\,(m_oa)\,B_o^1 + \sum_{q=1}^{\infty} t_q\,N_q^{-1/2}K_1(m_qa)\,B_q^1\right] + \right.$$
$$=\left[\frac{1}{8}\left(\frac{a}{d}\right)A_o^1 + \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\,A_n^1\,\frac{I_2(n\pi a/d)}{I_1(n\pi a/d)}\right]\Big\} \qquad (3.12)$$

where

$$t_o = \left\{\left(1-\frac{h}{d}\right)\frac{sh(m_od)}{m_od} + \frac{ch(m_oh)-ch(m_od)}{(m_od)^2}\right\}$$
$$t_q = \left\{\left(1-\frac{h}{d}\right)\frac{\sin\,(m_qd)}{m_qd} - \frac{\cos\,(m_qh) - \cos\,(m_qd)}{(m_qd)^2}\right\} \qquad (3.11)$$

$$\frac{a_{61}}{\rho Da} + i\,\frac{b_{61}}{\omega\rho Da} = \left(\frac{d}{a}\right)^2\left\{B_o^1\;H_1\,(m_oa)\,N_o^{-1/2}\,\frac{shm_oh-shm_od}{m_o(h-d)} + \sum_q B_q^1\,K_1(m_qr)N_q^{-1/2}\cdot\right.$$
$$\frac{\sin m_qh - \sin m_qd}{m_a(h-d)} \qquad (3.13)$$

4. LIMITING VALUES FOR HEAVE ADDED MASS AND DAMPING COEFFICIENTS FOR $\omega = 0$.

As the frequency of the motion ω approaches to zero the values of m_o given by (2.22) goes to zero as well while $m_q h$ values tend to $q\pi$. The coefficients of the linear equation (2.36) become real and the formulation correspond to the case where rigid boundary conditions exist at the free surface. The only imaginary value is the imaginary part of A_o which is equal to

$$\text{Im } A_o = A_{io} = \frac{\pi a^2}{2dh} \tag{4.1}$$

From which using 3.9 the damping coefficient is immediately obtained as:

$$\lim_{\omega \to o} \frac{b_{22}}{\omega \rho D} = \frac{\pi a^2}{4h(h-d)} \tag{4.2}$$

The above result can be also obtained by using the Haskinds relations given for finite depth by Newmann (1976). The equation (2.36), (2.37) and (2.38) for heave motion are

$$A_o^o = -\sum_{j=1}^{\infty} \gamma_{oj} A_j^o + h_o^o \tag{4.3}$$

$$\sum_{j=1}^{\infty} \gamma_{nj}^o \; A_j^o = h_n^o \tag{4.4}$$

$$\gamma_{nj}^o = \left\{ \left[\frac{H_o(m_o a)}{H_o'(m_o a)} \cdot \frac{E_{on} \cdot E_{oj}}{m_o h} + \sum_{q=1}^{\infty} \frac{K_o(m_j a)}{K_o'(m_j a)} \cdot \frac{E_{qn} \cdot E_{qj}}{m_q h} \right] U_j^o - S_{nj} \right\} \tag{4.5}$$

$$h_o^o = \left\{ \frac{H_o(m_o a)}{H_o'(m_o a)} \cdot \frac{\beta_o^o E_{oo}}{m_o h} + \sum_{q=1}^{\infty} \frac{\beta_q E_{qo}}{m_g h} - \left(\frac{1}{3} - \frac{1}{2}\left(\frac{a}{d}\right)^2\right) \right\} \tag{4.6}$$

$$h_n^o = \left\{ \frac{H_o(m_o a)}{H_o'(m_o a)} \cdot \frac{\beta_o^o E_{on}}{m_o h} + \sum_{q=1}^{\infty} \frac{\beta_q^o E_{qn}}{m_q h} \right\} - \frac{2(-1)^n}{(n\pi)^2} \tag{4.7}$$

The above equations for very small m_o values can be written as follows

$$\gamma_{oj} = \alpha(0,j) + i\, m_o^{\,2} \beta\,(0,j) \tag{4.8}$$

$$\gamma_{nj} = \alpha(n,j) + i\, m_o^{\,4}\, \beta(n,j) \tag{4.9}$$

where

$$\alpha\,(n,j) = \{\, \delta_{nj} + \frac{4jd^2h^2}{\pi^2}\; \frac{I_1(j\pi a/d)}{I_o\,(j\pi a/d)}\; (-1)^{j-n} \sum_{q=1}^{\infty} \frac{K_o(q\pi a/h)}{K_1(q\pi a/h)} \cdot$$

$$\frac{q\,(\sin\,(q\pi d/h))^2}{[(q\pi)^2 - (nh)^2]\,[(qd)^2 - (jh)^2]}\,\} \tag{4.10}$$

$$h_o = \{\frac{2}{\pi^3}\; \frac{ah^2}{d^3} \sum_{q=1}^{\infty} \frac{K_o(q\pi\, a/h)}{K_1(q\pi a/h)} \cdot \frac{1}{q^3}\, (\sin\, q\,\pi\, d/h)^2 - (\frac{1}{3}\frac{1}{2}(\frac{a}{d})^2)\} - i\{\frac{1}{2}\,\frac{\pi a^2}{dh}\} \tag{4.10}$$

$$h_n = \{\, \frac{2ah^2}{\pi^3 d}\, (-1)^n \sum_{q=1}^{\infty} \frac{K_o(q\pi a/h)}{K_1(q\pi a/h)} \cdot \frac{1}{q}\, \frac{(\sin\, q\pi d/h)^2}{[(qd)^2-(n\pi)^2]} - \frac{2(-1)^n}{(n\pi)^2}\} + i\,\{\frac{(-1)^n a^2 dm_o^{\,2}}{2\pi hn^2}\} \tag{4.11}$$

After these preparations taking the limit of (4.4) for very small m_o values and using (4.8), (4.9), (4.11) we obtain

$$\sum_{j=1}^{\infty} (A_{r_j} + i\, A_{i_j})\, [\alpha\,(n,j) + i\, m_o^4\, \beta\,(n,j)] = h_{rn} + i\, m_o^2\, h_{in} \tag{4.12}$$

Here subscript r and i refer to the real and imaginary part of the values.

Separating into real and imaginary parts

$$\sum_{j=1}^{\infty} [A_{rj}\, m_o^4\, \beta\,(n,j) + A_{ij}\, \alpha\,(n,j)] = h_{in}\, m_o^2$$

$$\sum_{j=1}^{\infty} [A_{rj}\, \alpha(n,j) - A_{ij}\, m_o^4\, \beta(n,j)] = h_{rn}$$

Taking the limit of the right hand side of the first equations as $m_o \to o$ we obtain

$$\sum_{j=1}^{\infty} A_{rj}\, [\alpha\,(n,j) + m_o^4\, \beta\,(n,j)] = h_{rn}$$

and $\quad A_{ij}\, \alpha(n,j) = -\, m_o^4\, \beta\,(n,j)\, A_{rj}$

One can see therefore that all A_j' s, j = 1,2... have real values.

As a special case let's take a floating disc. This can be expressed as a limiting case where $d \to h$. One can show that for this case the equation 4.4 has a diagonal matrix and the unknown A_n' s are given by

$$A_n^{o} = - \frac{2(-1)^n}{(n\pi)^2} \qquad n = 1,2 \ldots$$

$$A_o^{o} = - (\frac{1}{3} - \frac{1}{2} (\frac{a}{d})^2) \qquad (4.13)$$

Since all sine terms in equation 4.12 are equal to zero. The added mass of the circular disc in heaving motion for $\omega \to o$ is obtained as

$$\lim_{\omega \to o} \frac{a_{22}}{a^3} = \rho\pi \left(\frac{h}{a}\right) \{ \frac{1}{3} + \frac{1}{8} \left(\frac{a}{h}\right) - \frac{4}{\pi^3} \left(\frac{h}{a}\right) \sum_{n=1}^{\infty} \frac{1}{n^3} \frac{I_1 (n\pi a/h)}{I_o (n\pi a/h)} \qquad (4.14)$$

In another limiting case where d goes to zero, again all the sine terms in equation 4.12 are equal to zero and the coefficient matrix becomes diagonal. In this case one can show that as ω and d go to zero the heave added mass in fact is infinite. According to this present theory one can conjecture that for d<h and at finite h the heave added mass remain finite at low frequencies but goes to infinity for infinite depth.

5. NUMERIC SOLUTION AND RESULTS.

The specific formulas used for the calculation of added mass and damping coefficients are given in the Appendix. The required routines for the calculation of Bessel functions and solution of linear equation are obtained from IMSL computer library. The computations were done at the U. S. Naval Academy. The results for heave were first compared to the results published by Garrison (1975) and Kritis (1979). The results are given in Figure 2. Added mass values obtained by this theory compared well with those of Kritis while Garrison's number are observed to be higher. The damping coefficients for heave are observed to be less than the values reported by Kritis (1979) and they were observed to agree better with those of Garrison.

The heave added mass and damping coefficients are also compared with the experimental data reported by McCormick, et al (1980).

Figure 3 shows experimental data and theoretical values computed for ω = 3 rad/sec. In this diagram added mass value is nondimensionalized by the total mass of the cylinder of height equal to the depth of water. This is expressed in the diagram as ANU/MH. The experimental data are observed to remain above the theoretical curve the best correlation is observed at about depth/radius values equal to 7.

Experimental and theoretical damping coefficients are compared in Figure 4. Experimental values remained above the theoretical calculation while showing similar trends. This discrepency can be possibly explained by the viscous damping neglected by the theory. The numerical results are also tested with those reported by J. Bai (1976) and the results for added mass and damping coefficients are observed to agree with in a derivation of 3 percent. In all calculations the infinite series are represented by 20 terms.

The heave added mass values for different water depths and draft values are given in Figures 5 to 11. The general behavior of the curves is that as the frequency increases the values remain constant. At zero frequency numerically at least the added mass values are observed to increase. The behavior of the curves at small frequency is discussed in section 4. Damping coefficients for heave are given in Figures 12 to 17. At high frequencies these values are observed to tend to zero while at zero frequency the values are finite at shallow depth and tend to approach zero as the depth increases. Higher values are observed to correspond to small draft to radius values. Deep water cases correspond to h/a = 20.

The sway calculations are compared to the results published by Bai, Yeung (1974) and are presented in Figure 19. This computation tends to follow the values computed by Bai while the values reported by Isshiki remained low especially at peak values. Figures 20 to 25 show the sway added mass values. Added mass values are observed to increase as draft to radius is increased at all depths. These values also remained finite at small frequencies. All curves are observed to have a local maximum at about $\nu = 1$. Damping coefficients for sway are equal to zero at zero frequencies and are observed to increase as the draft increases. The curves for damping coefficients are given in Figures 26 to 31. At high frequencies the curves show a decreasing slope and the maximum values are again observed at about $\nu = 1$. Pitch computations are first checked with those reported by Bai and Garrison and a good agreement is observed.

Pitch moment of inertias are presented in Figures 32 to 37. Except at very small drafts the curves have a very small slope. Inertia coefficients are observed to decrease as draft increases at shallow waters while in deep water higher coefficients correspond to higher drafts. Figures 38 to 43 give the pitch damping coefficients. At shallow water (h/a = 1 h/a = 3) high damping coefficients correspond to small drafts while at deep water (h/a = 20) high damping coefficients are observed to correspond to high drafts. A peculiar curve is seen in Figure 42 for T/A = 0.1 which suggests that the damping coefficient for very shallow disks increases as draft decreases even at moderately deep waters. Pitch induced sway added mass (a_{61}) and pitch induced sway damping coefficients b_{61} are given in igure 44-46. It is interesting to note that some of these values are in fact negative for low drafts at finite depths such as h/a = 3 but as the draft increases the values become positive.

CONCLUSION

The solution presented in this paper offers a quick calculation of the hydrodynamic coefficients for a simple vertical circular cylinder. The results are compared to some available experimental numerical results. The agreement with numerical results are observed to be satisfactory. The comparisons with the experimental showed that even though the trend is well represented the amplitudes are not. This is partially acceptable at least for the case of damping coefficients where the viscous resistance must be effective. The input variables are water depth, radius of cylinder and its draft. Well documented routines can be used for the calculation of special functions and the solution of the linear equations. The necessary formulas for computer application are also presented.

The hydrodynamic coefficient to study the motion of the cylinder are presented in graphical format. It is hoped that this will increase the efficiency of future designs and that the designer will be able to estimate these coefficients rather precisely for his computations.

The limit of the heave added mass value for zero frequency is also discussed. It is shown that this quantity remains finite for finite depth and goes to infinity for infinite water depth. Special formulations are seen to be required for this limiting case.

The present formulation is currently extended to cylinders with variable cross sections.

APPENDIX

FORMULAS FOR COMPUTER APPLICATIONS

A-Heave

The linear set of equations for complex coefficients

$$\gamma_{nj} A_j = h_n$$

is solved first. The expression for γ_{nj} and h_n are given below

$$\gamma_{nj} = (\delta_{nj} + 16\, U_j (L\, p_o(n,j) + \sum_{q=1}^{\infty} \frac{K_o(m_q a)}{K_1(m_q a)} p_q(n,j)) + i \frac{32\, U_j T p_o(n,j)}{\pi\, m_{oa}}$$

$$h_i = \frac{4a}{d} (L\, p_o(i,o) + \sum_{q=1}^{\infty} \frac{Ko(m_q a)}{K_1(m_q a)} p_q(i,o) - \frac{2(-1)^n}{(n\, \pi)^2} + i \left[\frac{8}{\pi m_o d} T p_o(n,o) \right]$$

where

$$U_j = \frac{j\pi}{2} \frac{I_1 \, (j\frac{\pi a}{d})}{I_o \, (j\frac{\pi a}{d})} \quad j = 1, 2, 3 \ldots$$

$$P_o(n,j) = \frac{(-1)^{n+j} (m_o d \; sh\, (m_o d))^2}{[(2m_o h) + sh(2m_o h)][(m_o d)^2 + (n\pi)^2][(m_o d)^2 + (j\pi)^2]}$$

$$P_q(n,j) = \frac{(-1)^{n+j} (m_q d \; \sin(m_q d))^2}{[(2m_q h) + \sin(2m_q h)][(m_q d)^2 - (n\pi)^2][(m_q d)^2 - (j\pi)^2]}$$

$$L = \frac{J_o(m_o a)\, J_1\, (m_o a) + Y_o(m_o a)\, Y_1(m_o a)}{[J_1(m_o a)]^2 + [Y_1(m_o a)]^2}$$

$$T = \frac{1}{[J_1(m_o a)]^2 + [Y_1(m_o a)]^2}$$

The added mass and damping coefficients are then computed by the following equation:

$$\frac{a_{22}}{\rho D} + i\,\frac{b_{22}}{\omega\rho D} = \frac{d}{h-d}\left\{\frac{1}{2} - \frac{1}{8}\left(\frac{a}{d}\right)^2 + \frac{1}{2}A_o + \frac{2}{\pi}\left(\frac{d}{a}\right)\sum_{n=1}^{\infty}\frac{A_n(-1)^n}{n}\,\frac{I_1\left(\frac{n\pi a}{d}\right)}{I_o\left(\frac{n\pi a}{d}\right)}\right\}$$

where a_{22}, b_{22} are the added mass and damping for heave, D is the volume of the cylinder in water.

B. Sway

The linear set of equations for sway can be written in the form:

$$\gamma_{nj}\,A_j = -\,h_n$$

where

$$\gamma_{nj} = \left\{-\delta_{nj} + 16U_j\left[L\cdot P_o(n,j) + \sum_{q=1}^{\infty} D_q\cdot P_q(n,j)\right]\right\} - i\frac{32}{\pi m_o a}\,U_j\cdot T\cdot P_o(n,j)$$

$$h_n = 8\frac{d}{a}\left\{L\cdot P_o(n,o)K_o + \sum_{k=1}^{\infty} D_q\cdot P_q(n,o)K_k\right\} - i\frac{16}{\pi m_o a}\,T\cdot P_o(n,o)K_o$$

and where

$$U_j = \left\{\frac{j\pi}{2}\cdot\frac{I_o\left(\frac{j\pi a}{d}\right)}{I_1\left(\frac{j\pi a}{d}\right)} - \frac{d}{2a}\right\} \text{ for } \quad = 1,2,\ldots \text{ and } U_o = \frac{d}{4a}$$

$$T = \Big([J_o^2(m_o a) + Y_o^2(m_o a)] + \frac{1}{(m_o a)^2}[J_1^2(m_o a) + Y_1^2(m_o a)] - \frac{2}{m_o a}$$
$$[J_o(m_o a)J_1(m_o a) + Y_o(m_o a)Y_1(m_o a)]\Big)^{-1}$$

$$L = \left\{[J_o(m_o a)J_1(m_o a) + Y_o(m_o a)\,Y_1(m_o a)] - \frac{1}{m_o a}[J_1^2(m_o a) + Y_1^2(m_o a)]\right\}T$$

$$K_o = \left\{\frac{sh(m_o h) - sh(m_o d)}{sh(m_o d)}\right\}, \quad K_q = \left\{\frac{\sin(m_q h) - \sin(m_q d)}{\sin(m_q d)}\right\}$$

$$D_k = -\frac{m_q a}{1 + m_q a\,\frac{K_o(m_q a)}{K_1(m_q a)}}, \quad q = 1, 2, 3, \ldots$$

The definition for p_o and p_q remain the same as in heave.

The sway added mass and damping coefficients are obtained as

$$\frac{a_{11}}{\rho D} + i\,\frac{b_{11}}{\omega\rho D} = -\,\{B_o H_1(m_o a)\,\frac{[sh(m_o h)-sh(m_o d)]}{N_o^{\frac{1}{2}}\,m_o(h-d)} + \sum_{q=1}^{\infty} B_q\;K_1(m_k a)\frac{[\sin(m_q h)-\sin(m_q d)]}{N_k^{\frac{1}{2}}\,m_q(h-d)}$$

where

$$B_q = \frac{1}{hm_q F_q'(a)}\,\{\frac{A_o}{4a}\,d\,E_{qo} + \sum_{n=1}^{\infty} A_n\,U_n\,E_{qn}\} + \frac{\beta_q'^{1}}{hm_q\;F'(a)}$$

$$n = o, 1, 2, \ldots \qquad q = o, 1, 2, \ldots$$

$$E_{on} = \frac{(-1)^n N_o^{-\frac{1}{2}} 2m_o d\; sh(m_o d)}{[(m_o d)^2 + (n\pi)^2]}$$

$$E_{qn} = \frac{(-1)^n N_q^{-\frac{1}{2}} 2m_q d\; \sin(m_q d)}{[(m_q d)^2 - (n\pi)^2]}$$

$$\beta_o{}^1 = \frac{1}{a}\,N_o^{-\frac{1}{2}}\,[\frac{sh\;m_o h - sh\;m_o d}{m_o}]\;,\quad \beta_q{}^1 = \frac{1}{a}\,N_q^{-\frac{1}{2}}\,[\frac{\sin(m_q h) - \sin(m_q d)}{m_q}]$$

and

$$N_o = \frac{1}{2}\,(1 + \frac{\sinh({}^2moh\;)}{{}^2moh}$$

$$N_q = \frac{1}{2}\,(1 + \frac{\sin(2m_q h)}{2m_o h}\;)$$

C. PITCH

The linear set of complexed valued equations is

$$\gamma_{nj} \quad A_j = - h_n \qquad n = o,1,2 \ , \ j = 0,\ 1,\ 2 \ \ldots$$

where

$$\gamma_n = \{ - \delta_{nj} + 16\ Uj\ [L \cdot P_o(n,j) + \sum_{q=1}^{\infty} D_q \cdot P_q(n,j)]\} - i\ \frac{32}{\pi m_o a} U_j \cdot T \cdot P_o(n,j)$$

$$h_n = 8\ \{L \cdot P_o(n,o) \cdot \Theta_o + \sum_{q=1}^{\infty} D_q \cdot P_q(n,o) \cdot \Theta_k\} - A_n - i\ \frac{16}{\pi m_o a} \cdot T \cdot P_o(n,o) \quad \Theta_o$$

$$U_j = \{\frac{j\pi}{2} \ \frac{I_o(\ r a/d)}{I_1(\ r a/d)}\} \text{ for } j=1,2,\ldots \quad U_o = \frac{d}{4a} \quad , \delta_{nj} = \{\begin{matrix} 0 & n \neq j \\ 1 & n = j \end{matrix}$$

$$\Theta_o = \{[\frac{3}{2} - \frac{h}{d} - \frac{3}{8}(\frac{a}{d})^2 + \frac{1}{(m_o d)^2}] + \frac{ch(m_o h) - 2ch(m_o d)}{(m_o d) sh(m_o d)}\}$$

$$\Theta_q = \{[\frac{3}{2} - \frac{h}{d} - \frac{3}{8}(\frac{a}{d})^2 - \frac{1}{(m_q d)^2}] - \frac{\cos(m_q h) - 2ch(m_q d)}{(m_q d)\ \sin\ (m_q d)}\}$$

$$\alpha_n{}^1 = \{ \frac{2(-1)^n}{(n\pi)^2} (\frac{a}{d}) \} \text{ for } n = 1,\ 2,\ 3,\ \ldots \quad \alpha_o{}^1 = \{ \frac{1}{3} (\frac{a}{d}) - \frac{1}{4} (\frac{a}{d})^3 \} \text{ for } n = o$$

The expressions for P_o, P_q, L, T, D_q remain the same as defined for sway.

The roll (Pitch) added moment of inertia is calculated as

$$\frac{a_{66} + i\ b_{66}/\omega}{\rho D\ a^2} = - \frac{d}{h-d} \ \{ \frac{d}{a} [\frac{t_o}{N_o} \frac{H_1}{1/2} (m_o r)\ B_o + \sum_{q=1}^{\infty} \frac{t_q}{N_q} \frac{K_1}{1/2} (m_q a)\ B_q\ ] +$$

$$- [\ \frac{1}{8} (\frac{a}{d})^2 (1 - \frac{1}{6} (\frac{a}{d})^2) + \frac{1}{8}(\frac{a}{d})\ A_o + \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \frac{I_2(\frac{n\pi a}{d})}{I_1(\frac{n r a}{d})} A_n]\}$$

The expression for B_o, B_q remain the same except the B* values for roll are given by

$$\beta_o^{1} = N_o^{-\frac{1}{2}} \{ [\frac{3}{2} - \frac{h}{d} - \frac{3}{8}(\frac{a}{d})^2 + \frac{1}{(m_o d)^2}] \frac{sh(m_o d)}{(m_o d)} + \frac{ch(m_o h) - 2ch(m_o d)}{(m_o d)^2} \}$$

$$\beta_q^{1} = N_q^{-\frac{1}{2}} \{ [\frac{3}{2} - \frac{h}{d} - \frac{3}{8}(\frac{a}{d})^2 - \frac{1}{(m_q d)^2}] \frac{\sin(m_q d)}{(m_q d)} - \frac{\cos(m_q h) - 2\cos(m_q d)}{(m_q d)^2} \}$$

and the expressions for t_o and t_q are

$$t_o = \{ (1 - \frac{h}{d}) \frac{sh(m_o d)}{m_o d} + \frac{ch(m_o h) - ch(m_o d)}{(m_o d)^2} \}$$

$$t_q = \{ (1 - \frac{h}{d}) \frac{\sin(m_q d)}{m_q d} - \frac{\cos(m_q h) - \cos(m_q d)}{(m_q d)^2} \}$$

REFERENCES

Abramowitz, M.; Stegun, I.A., (1964) "Handbook of Mathematical Functions." National Bureau of Standards, Washington, D. C.

Bai, J., (1976) "The Added Mass and Damping Coefficients of and the Excitation Forces on Four Axisymmetric Ocean Platforms." Naval Ship Research and Development Center, report no. SPD-670-01, April

Bai, K. J.; Yeung, R. W. (1974), "Numerical Solutions to Free-Surface Flow Problems." Tenth Symposium Naval Hydrodynamics, pp. 609-647, Cambridge, Mass.

Garrison, C. J., (1975) "Hydrodynamics of Large Objects in the Sea, Part II. Motion of Free-Floating Bodies." Journal of Hydronautics, vol. 9, No. 2, pp. 58-63, April.

Garrett, C. J., (1971) "Wave Forces on a Circular Dock." J. Fluid Mech., vol. 46, part 1, pp. 129-139.

Isshiki, H. and Hwang, J. H., "An Axi-Symmetric Dock In Waves," Seoul National University, Lorea, College of Engineering, Dept. of Naval Architecture, Report No. 73-1, Jan. 1973, 38 pp.

Kim, W. J., (1974) "On the Harmonic Oscillations of a Rigid Body on a Free Surface." Journal of Fluid Mechanics, vol. 21, pp. 427-451.

Kritis, Ir. B., (1979) "Heaving Motions of Axisymmetric Bodies." The Naval Architect, January, pp. 26-26.

MacCamy, R. C., (1961) "On the Heaving Motion of Cylinders of Shallow Draft." Journal of Ship Research, December, Volume 5, No. 3.

McCormick, M. E.; Coffey, J. P.; Richardson, J. B., (1980) An Experimental Study of Wave Power Conversion by a Heaving Vertical Circular Cylinder in Restricted Waters," U. S. Naval Academy, Engineering Report EW 10-80, March.

Newman, J. N., (1975) "Marine Hydrodynamics." The MIT Press, Cambridge, Massachusetts and London, England.

Newman, J. N., (1976), "The Interaction of Stationary Vessels With Regular Waves." Eleventh Symposium Naval Hydrodynamics, pp. 491-501, London, England.

Wang, Shen, (1966) "The Hydrodynamic Forces and Pressure Distributions for an Oscillating Sphere in a Fluid of Finite Depth." MIT Department of Naval Architecture and Marine Engineering, Doctoral Thesis, June.

Yeung, R. W., (1975) "A Hybrid Integral-Equation Method for Time Harmonic Free Surface Flow." 1st. Int. Conf. Numer. Ship Hydrodynamics, Gaithersburg, Maryland, pp. 581-608.

ACKNOWLEDGEMENT

Professor T. Sabuncu would like to thank the Council for International Exchange of Scholars for the support he received through his stay at MIT as a visiting professor. Thanks are also due to Professors J. N. Newman, R. Young, F. Noblesse, of MIT for their interest, discussions and to Professor P. F. Wiggins of the U. S. Naval Academy whose interest made the computer application possible.

Special thanks are also extended to Ms. Virginia Christensen and Miss Sharon Vaughn for typing this report.

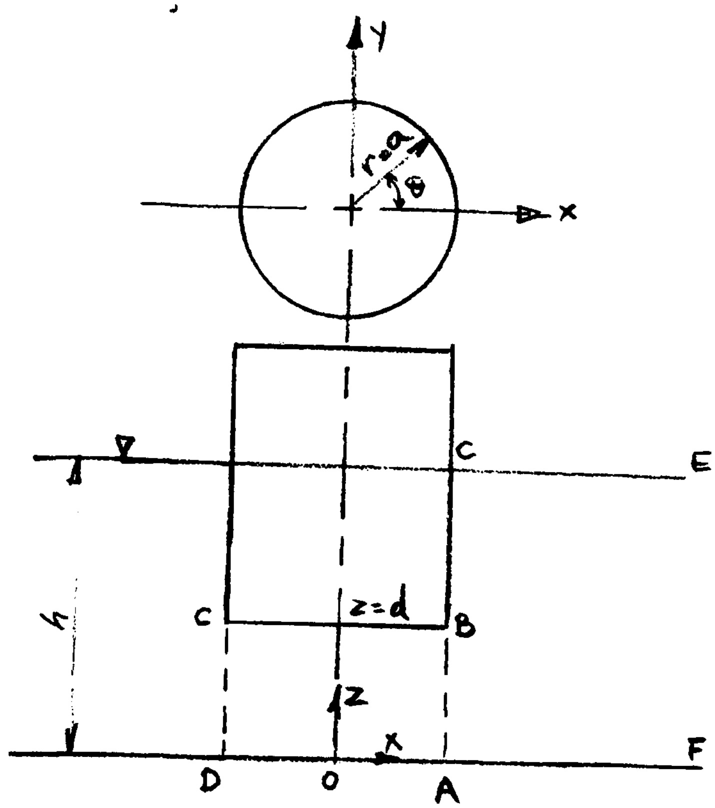


Figure 1

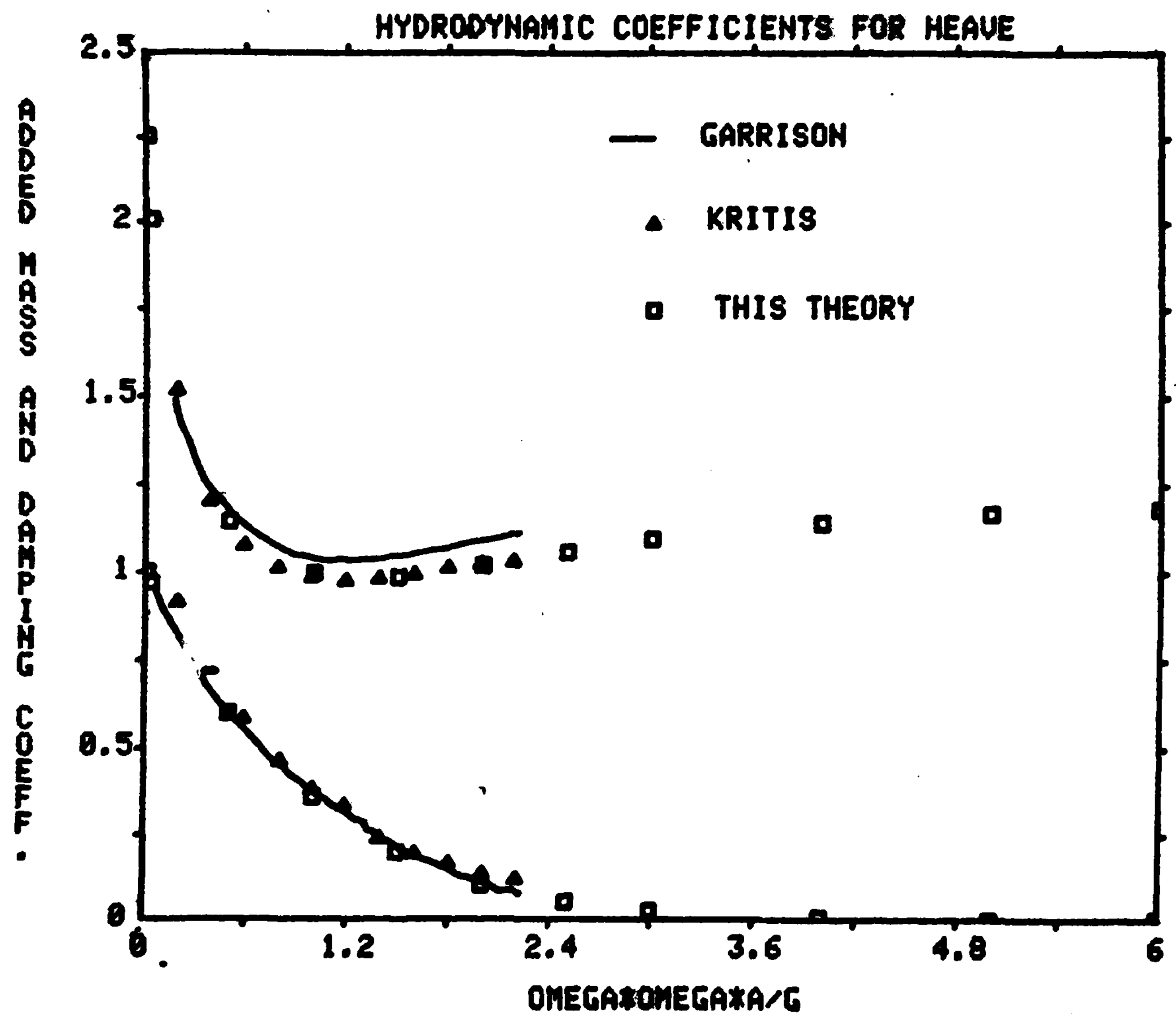

HYDRODYNAMIC COEFFICIENTS FOR HEAVE
GARRISON
KRITIS
THIS THEORY
ADDED MASS AND DAMPING COEFF.
2.5
2
1.5
1
0.5
0
1.2
2.4
3.6
4.8
6
OMEGA*OMEGA*A/G

Figure 2

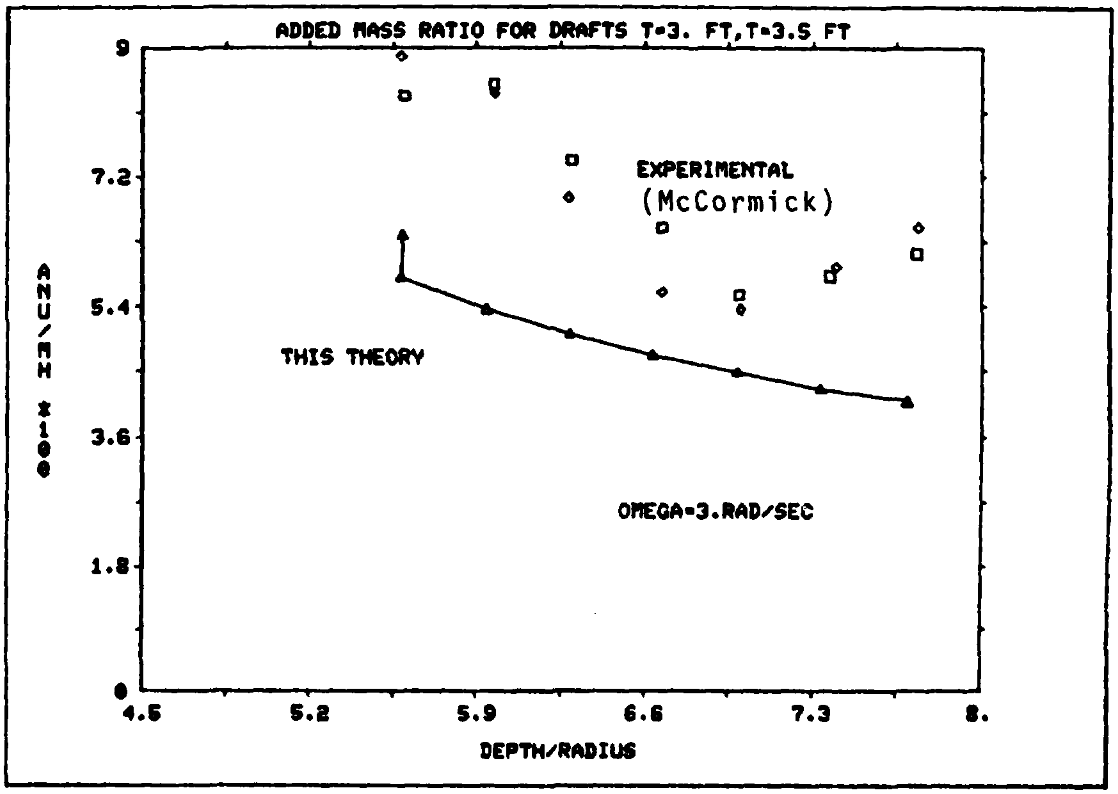

ADDED MASS RATIO FOR DRAFTS T=3. FT,T=3.5 FT
EXPERIMENTAL
(McCormick)
THIS THEORY
OMEGA=3.RAD/SEC
ANU/MH *100
DEPTH/RADIUS
9
7.2
5.4
3.6
1.8
0
4.5
5.2
5.9
6.6
7.3
8.

Figure 3

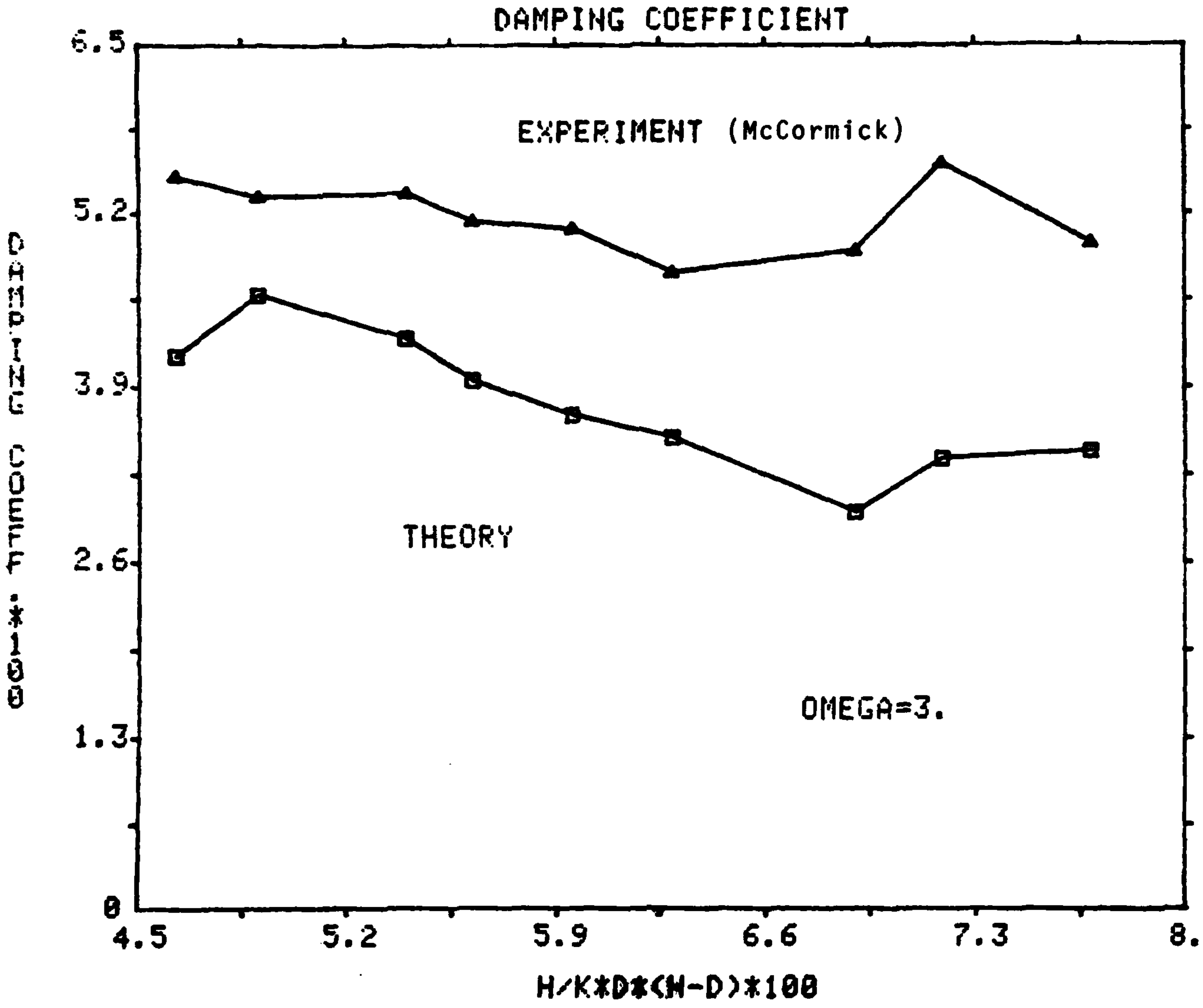

DAMPING COEFFICIENT
EXPERIMENT (McCormick)
THEORY
OMEGA=3.
6.5
5.2
3.9
2.6
1.3
0
4.5
5.2
5.9
6.6
7.3
8.
H/K*D*(H-D)*100
DAMPING COEFF.*100

Figure 4

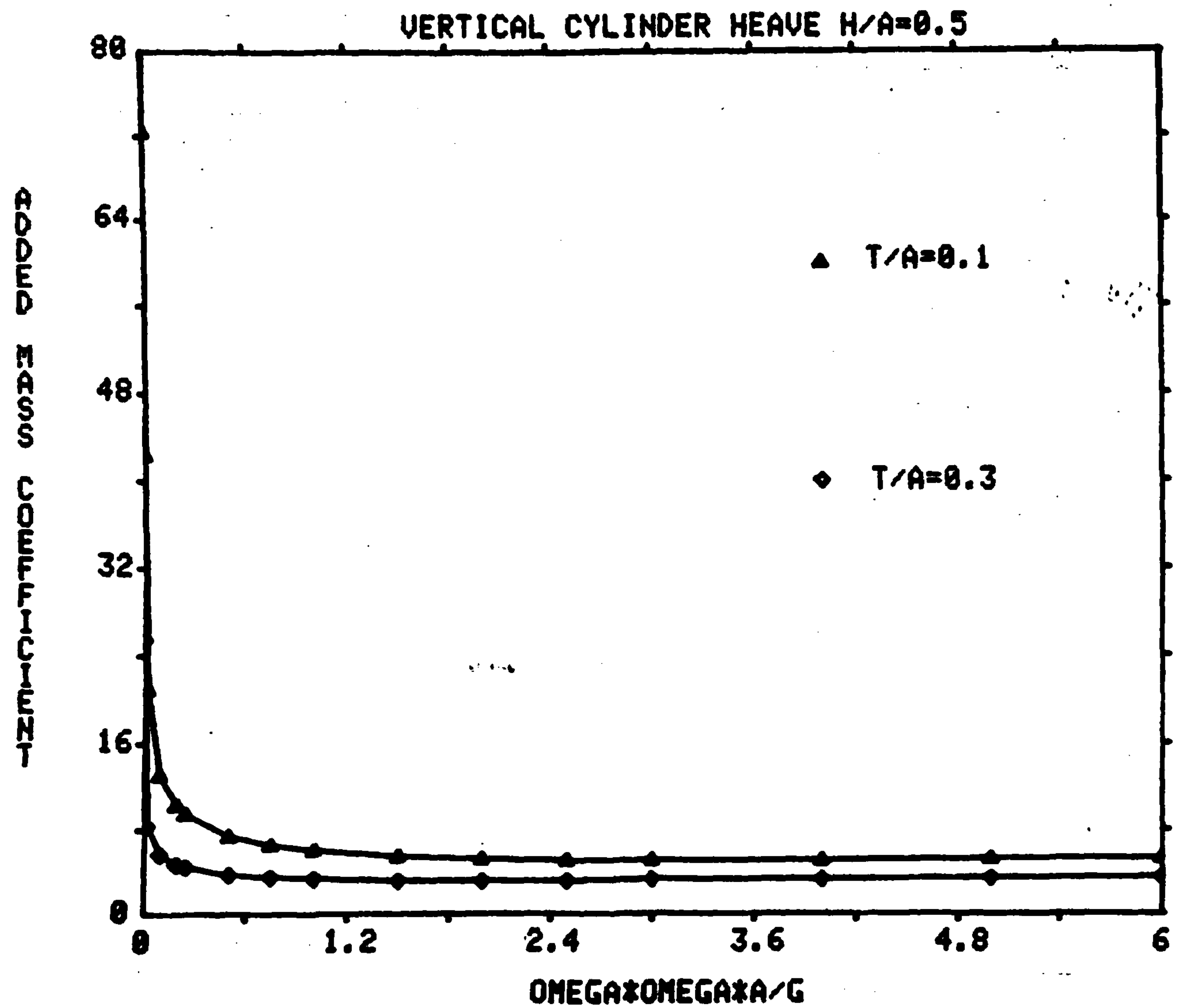

VERTICAL CYLINDER HEAVE H/A=0.5
T/A=0.1
T/A=0.3
ADDED MASS COEFFICIENT
OMEGA*OMEGA*A/G
80
64
48
32
16
0
1.2
2.4
3.6
4.8
6

Figure 5

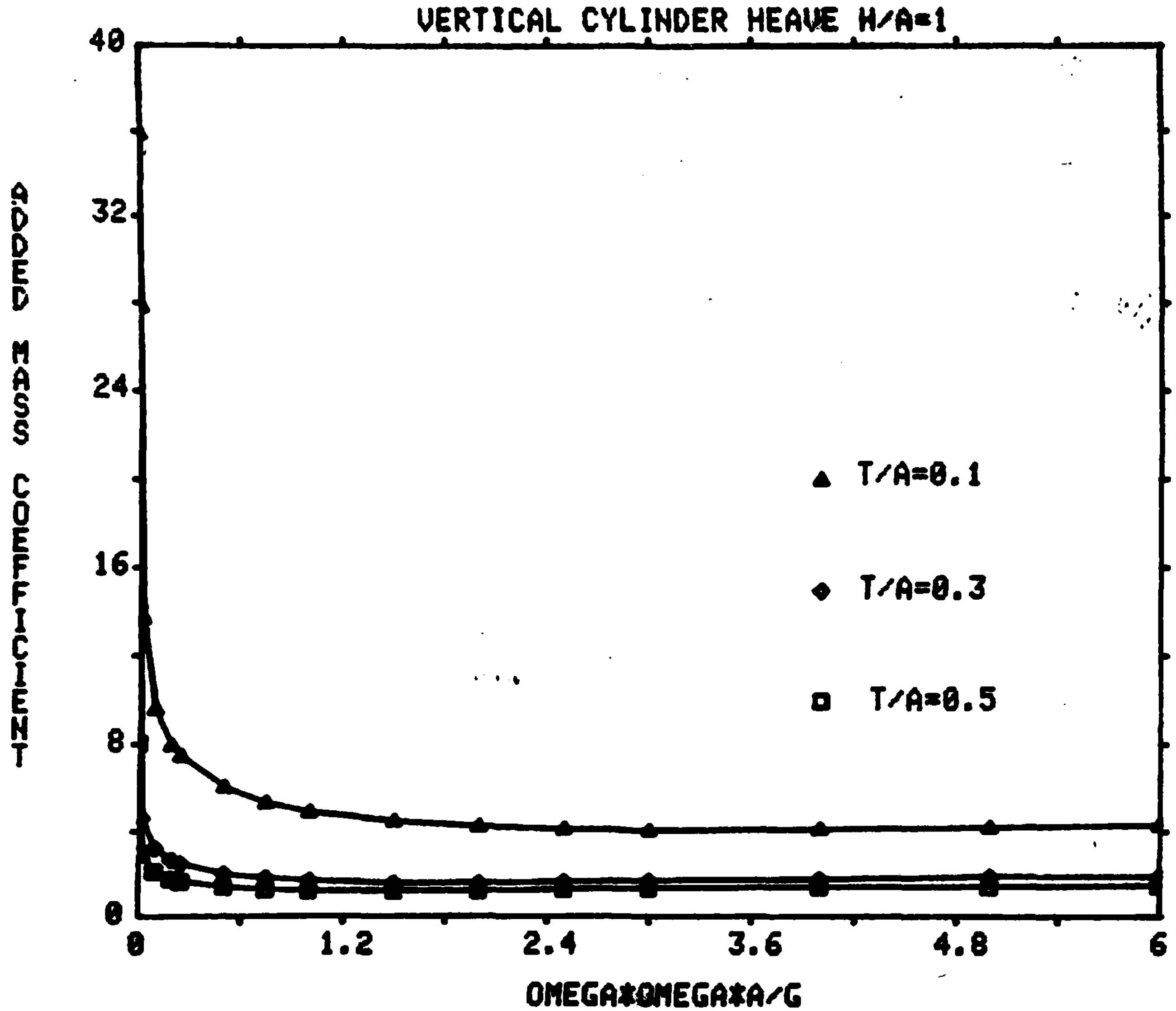

VERTICAL CYLINDER HEAVE H/A=1
40
32
24
16
8
0
ADDED MASS COEFFICIENT
0
1.2
2.4
3.6
4.8
6
OMEGA*OMEGA*A/G
T/A=0.1
T/A=0.3
T/A=0.5

Figure 6

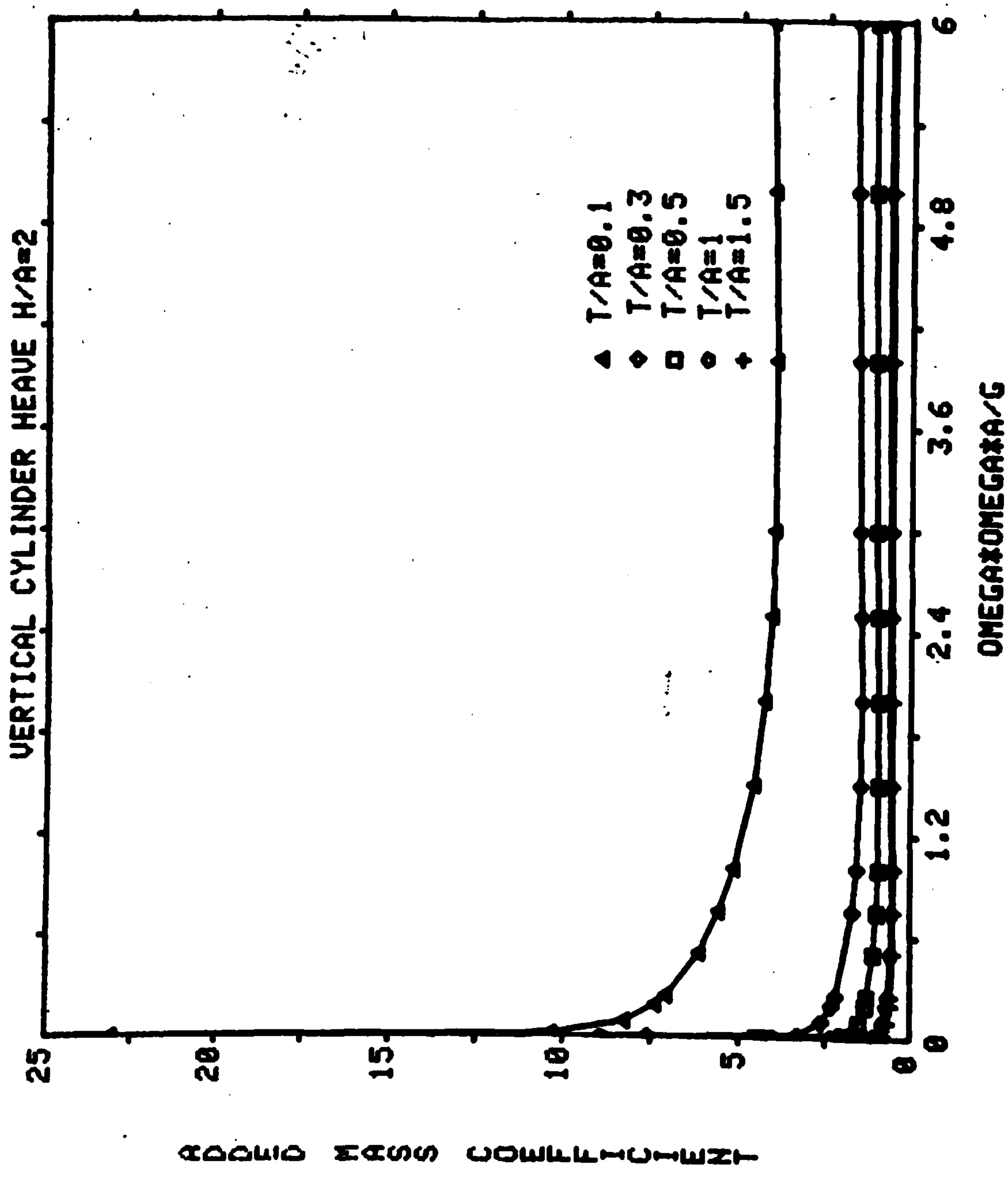

VERTICAL CYLINDER HEAVE H/A=2
T/A=0.1
T/A=0.3
T/A=0.5
T/A=1
T/A=1.5
ADDED MASS COEFFICIENT
OMEGA*OMEGA*A/G
0
1.2
2.4
3.6
4.8
6
0
5
10
15
20
25

Figure 7

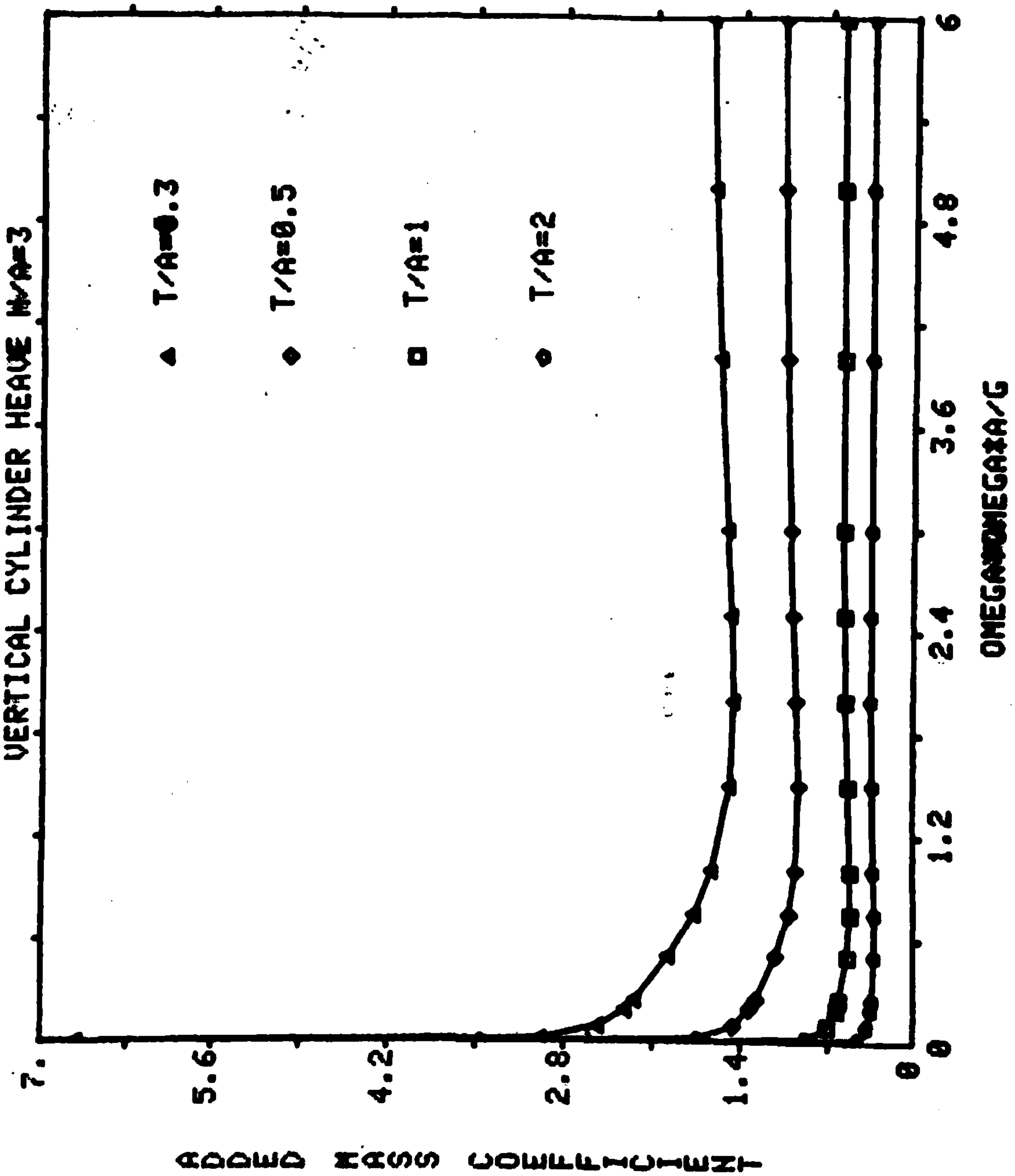
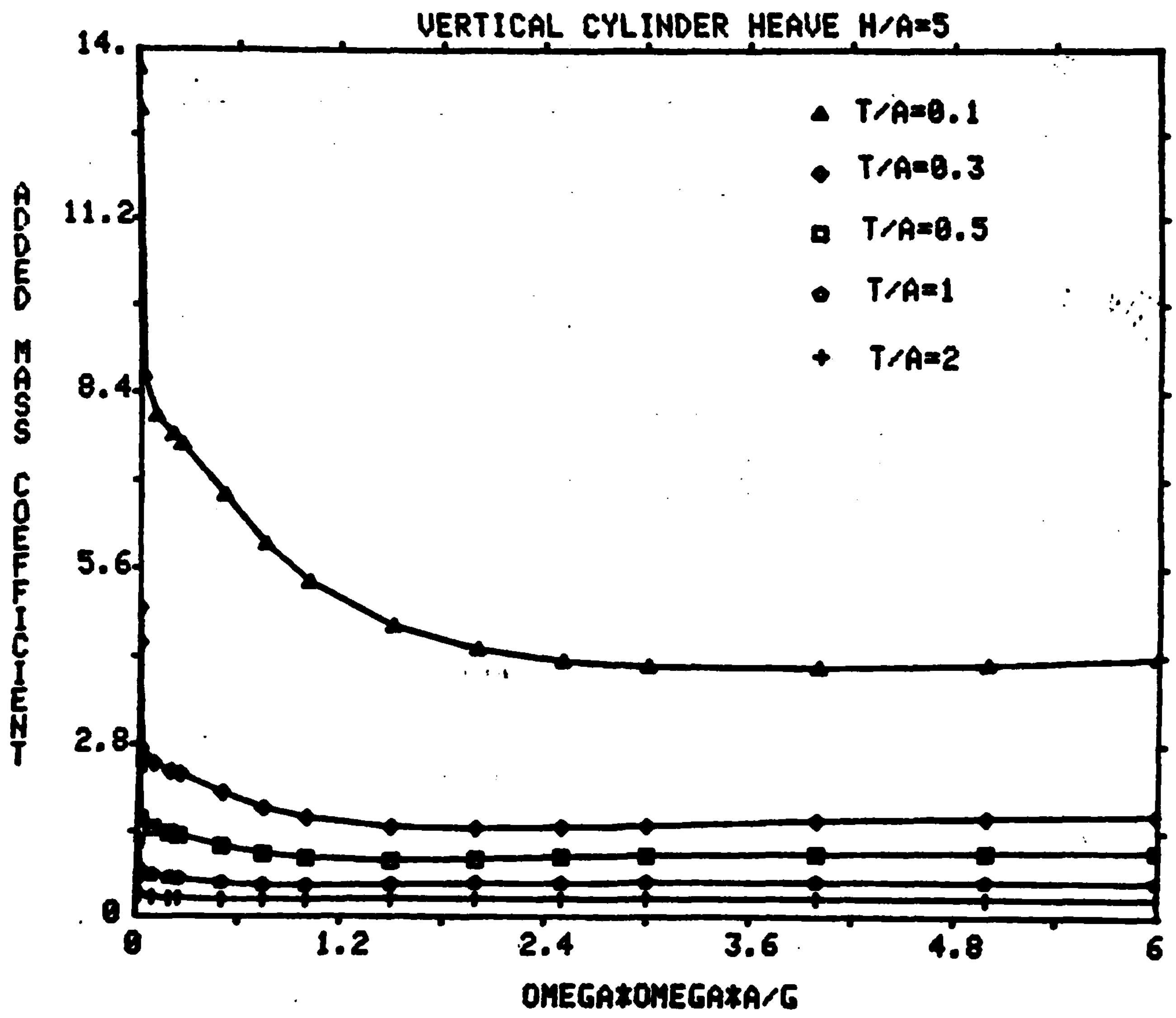
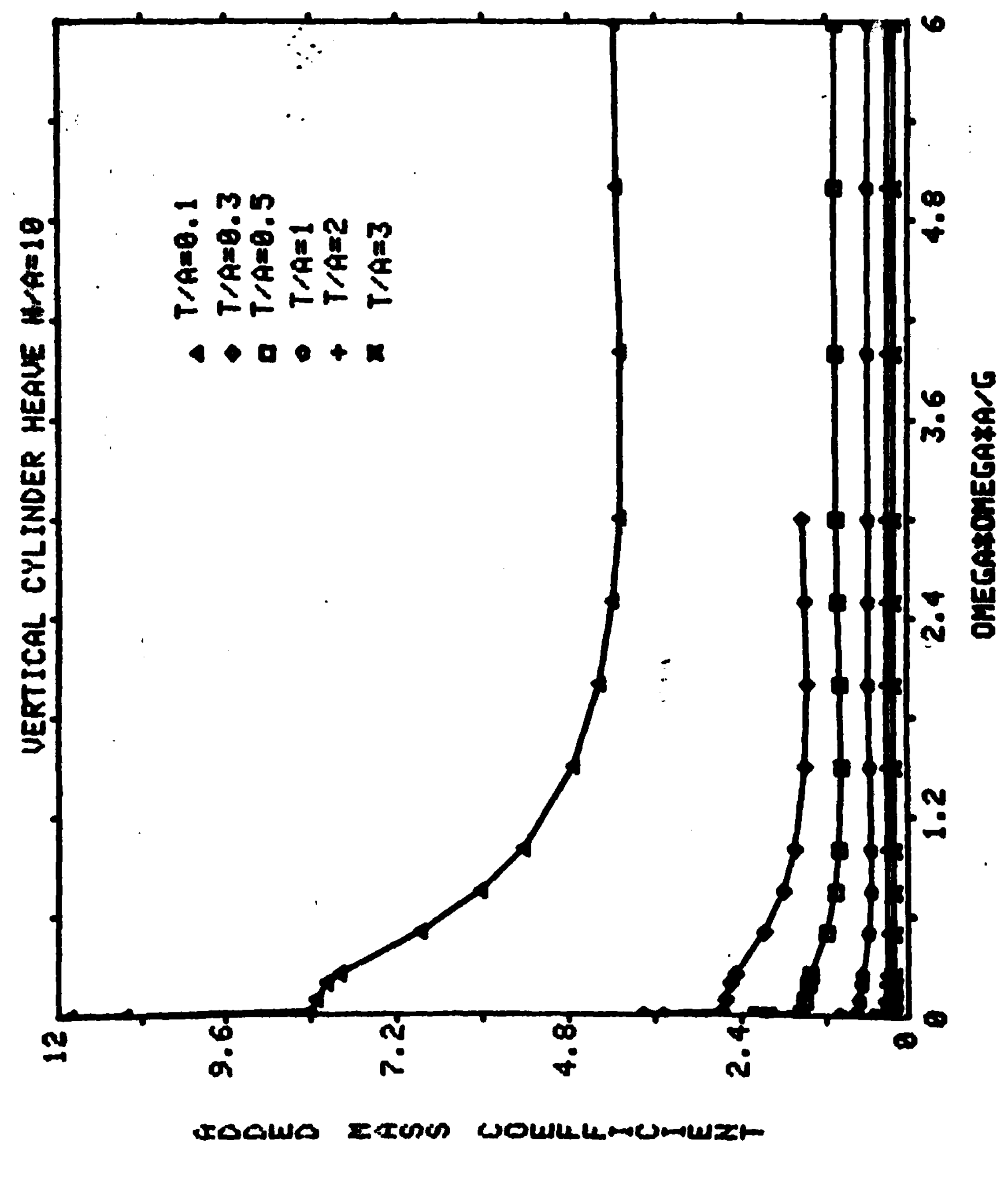

VERTICAL CYLINDER HEAVE H/A=3
T/A=0.3
T/A=0.5
T/A=1
T/A=2
7.
5.6
4.2
2.8
1.4
0
0
1.2
2.4
3.6
4.8
6
OMEGA*OMEGA*A/G
ADDED MASS COEFFICIENT

Figure 8



VERTICAL CYLINDER HEAVE H/A=5
T/A=0.1
T/A=0.3
T/A=0.5
T/A=1
T/A=2
ADDED MASS COEFFICIENT
14.
11.2
8.4
5.6
2.8
0
1.2
2.4
3.6
4.8
6
OMEGA*OMEGA*A/G

Figure 9



VERTICAL CYLINDER HEAVE H/A=10
T/A=0.1
T/A=0.3
T/A=0.5
T/A=1
T/A=2
T/A=3
ADDED MASS COEFFICIENT
12
9.6
7.2
4.8
2.4
0
0
1.2
2.4
3.6
4.8
6
OMEGA*OMEGA*A/G

Figure 10

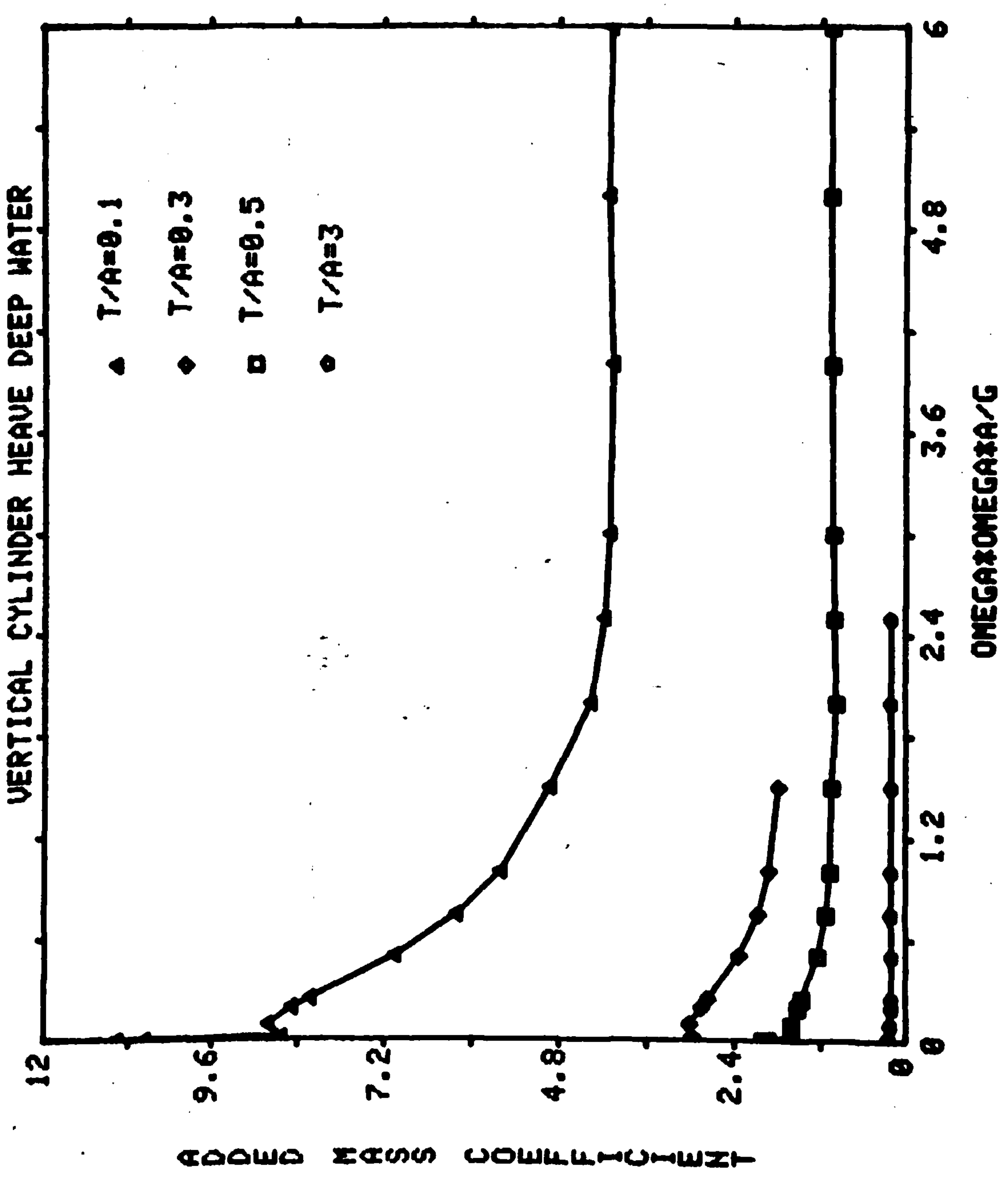

VERTICAL CYLINDER HEAVE DEEP WATER
T/A=0.1
T/A=0.3
T/A=0.5
T/A=3
ADDED MASS COEFFICIENT
12
9.6
7.2
4.8
2.4
0
OMEGA*OMEGA*A/G
0
1.2
2.4
3.6
4.8
6

Figure 11

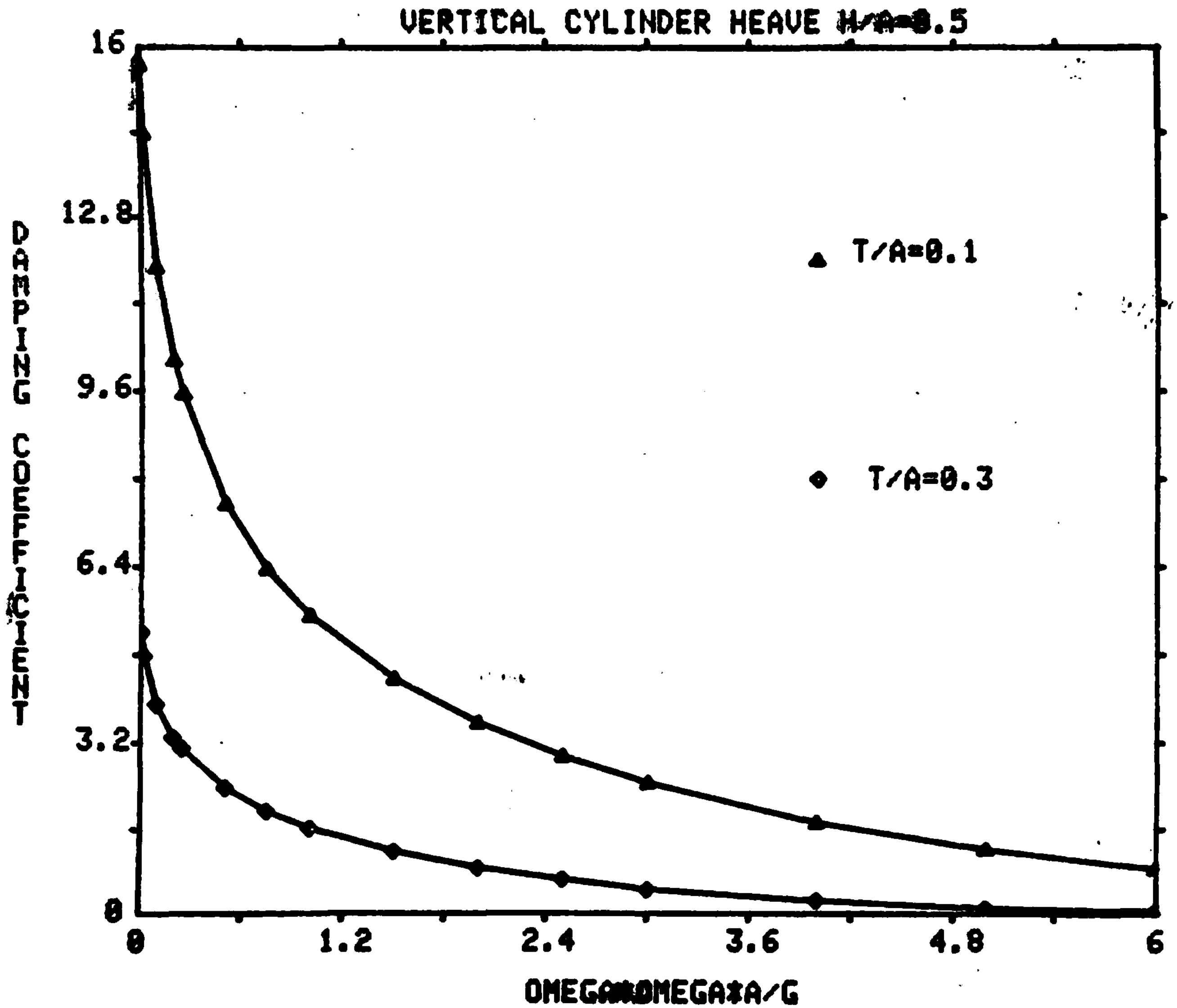

VERTICAL CYLINDER HEAVE H/A=0.5
DAMPING COEFFICIENT
16
12.8
9.6
6.4
3.2
0
1.2
2.4
3.6
4.8
6
T/A=0.1
T/A=0.3
OMEGA*OMEGA*A/G

Figure 12

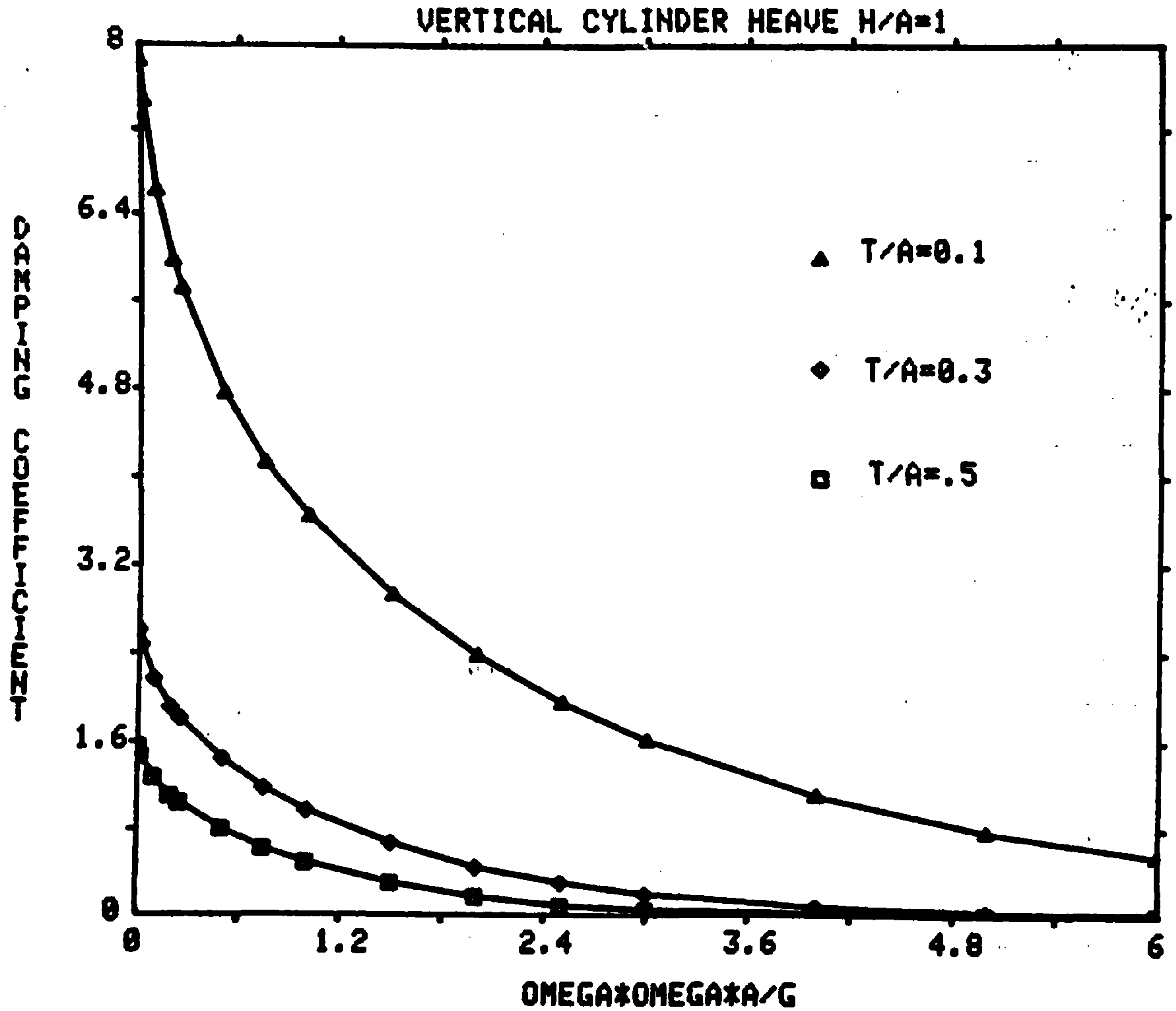

VERTICAL CYLINDER HEAVE H/A=1
DAMPING COEFFICIENT
8
6.4
4.8
3.2
1.6
0
1.2
2.4
3.6
4.8
6
OMEGA*OMEGA*A/G
T/A=0.1
T/A=0.3
T/A=.5

Figure 13

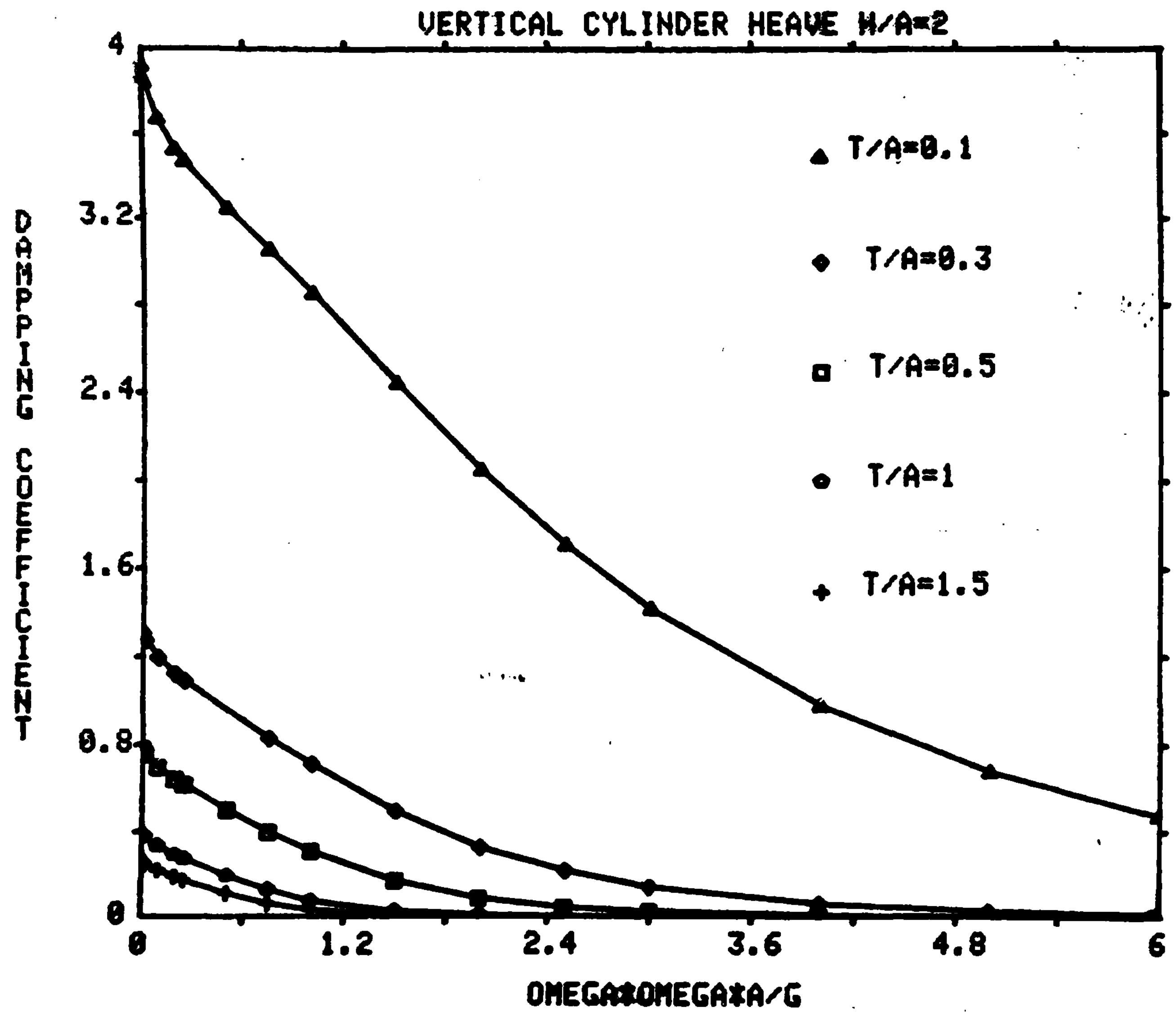
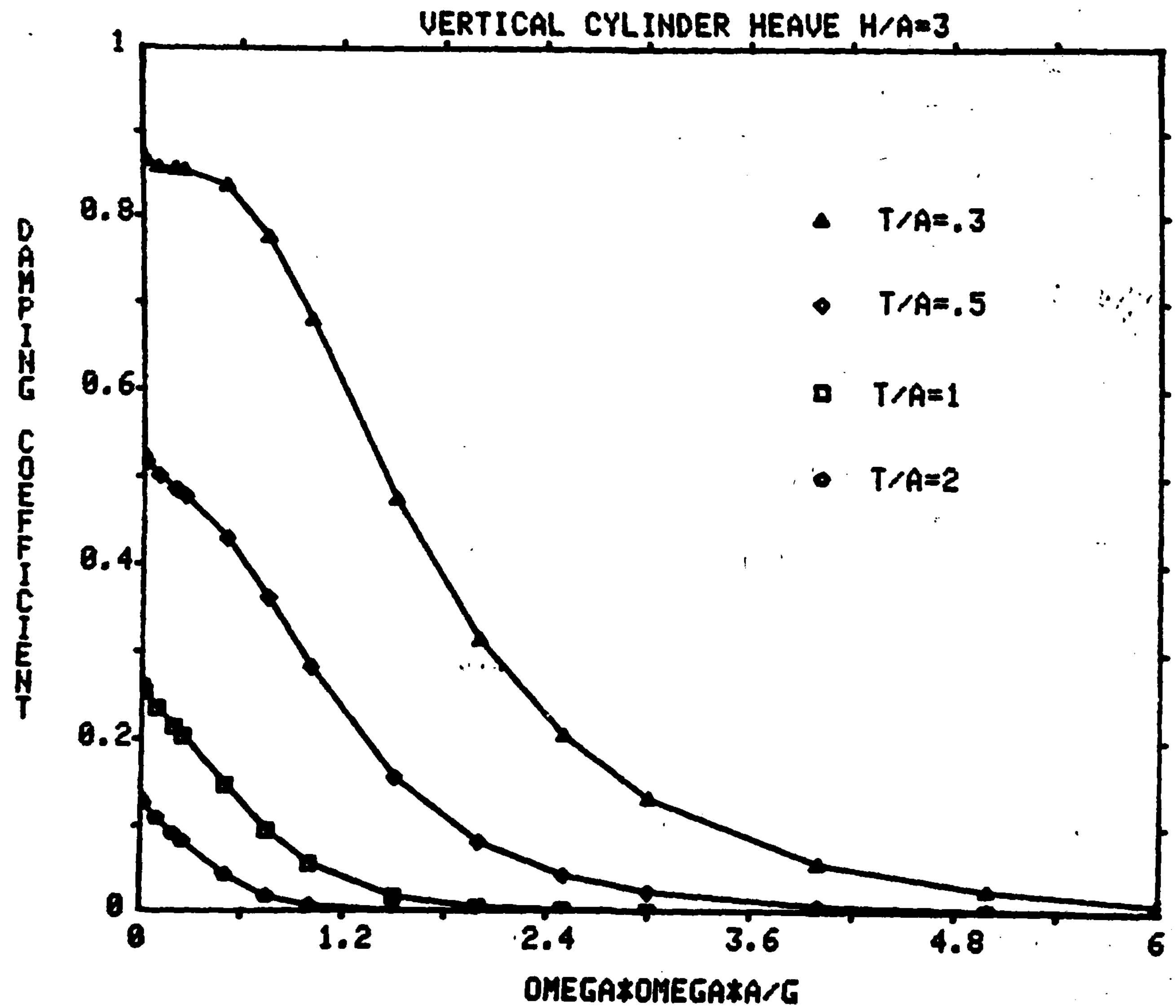

VERTICAL CYLINDER HEAVE H/A=2
T/A=0.1
T/A=0.3
T/A=0.5
T/A=1
T/A=1.5
DAMPING COEFFICIENT
OMEGA*OMEGA*A/G
0
0.8
1.6
2.4
3.2
4
1.2
2.4
3.6
4.8
6

Figure 14



VERTICAL CYLINDER HEAVE H/A=3
DAMPING COEFFICIENT
OMEGA*OMEGA*A/G
T/A=.3
T/A=.5
T/A=1
T/A=2
1
0.8
0.6
0.4
0.2
0
1.2
2.4
3.6
4.8
6

Figure 15

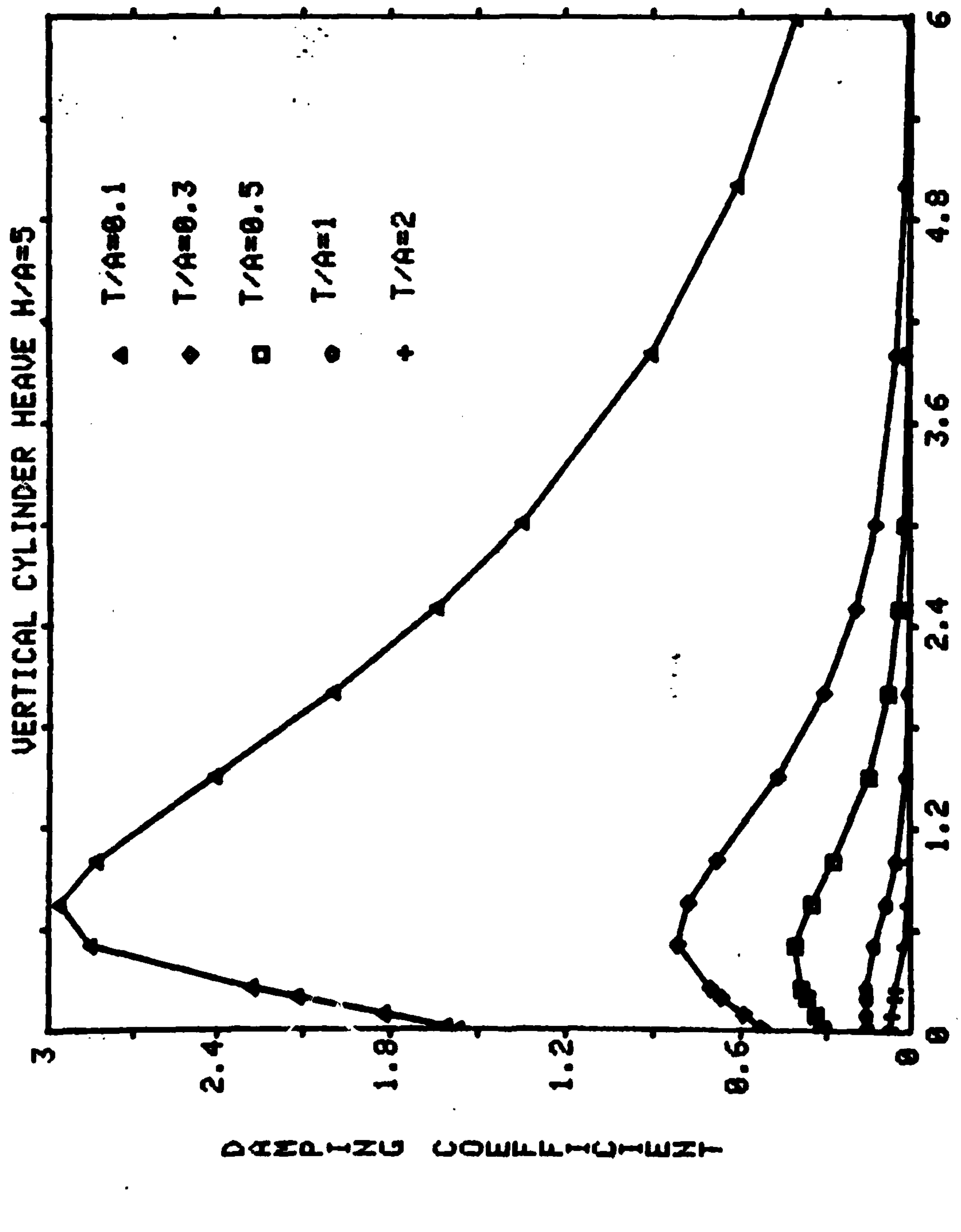

VERTICAL CYLINDER HEAVE H/A=5
T/A=0.1
T/A=0.3
T/A=0.5
T/A=1
T/A=2
DAMPING COEFFICIENT
OMEGA*OMEGA*A/G
0
0.6
1.2
1.8
2.4
3
0
1.2
2.4
3.6
4.8
6

Figure 16

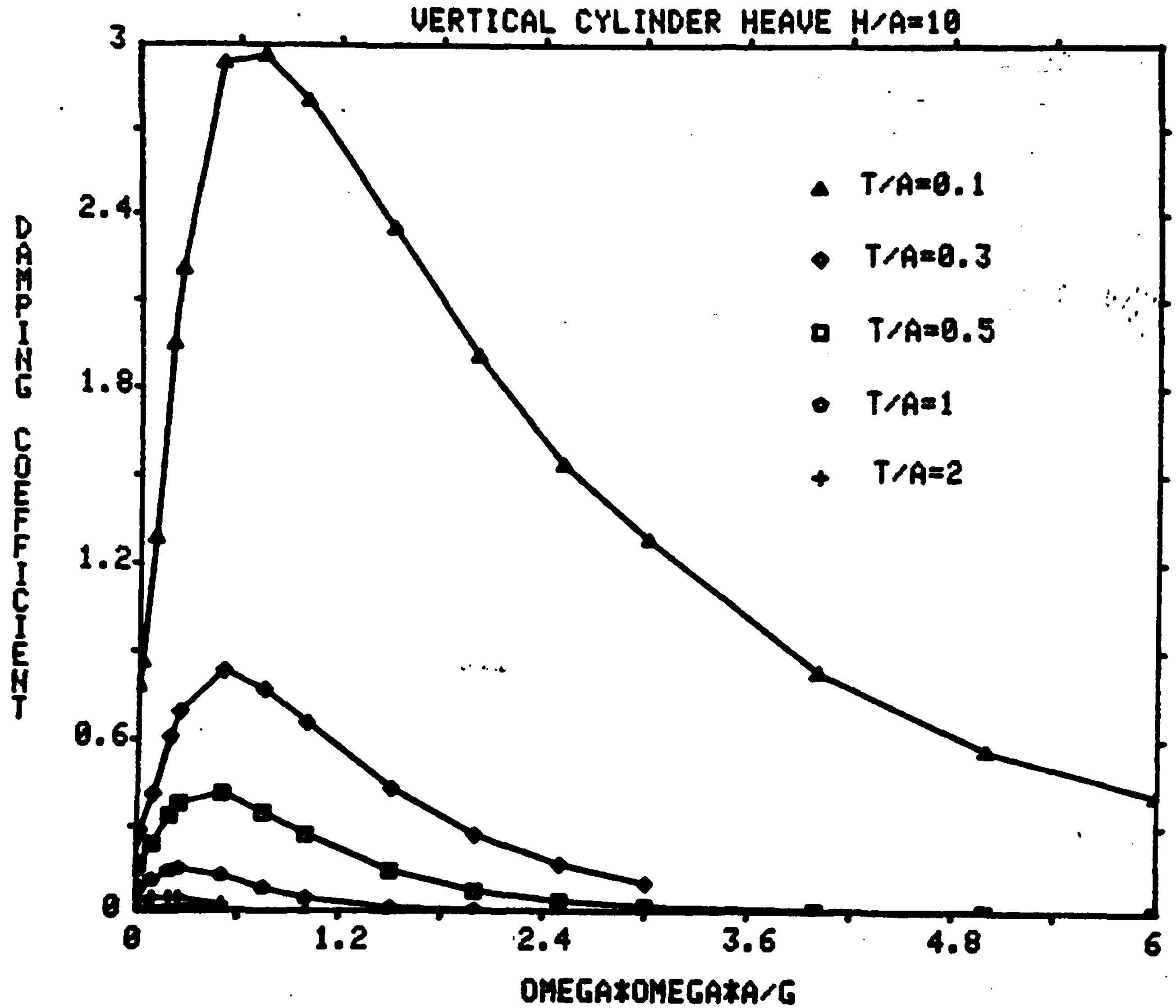

VERTICAL CYLINDER HEAVE H/A=10
T/A=0.1
T/A=0.3
T/A=0.5
T/A=1
T/A=2
DAMPING COEFFICIENT
OMEGA*OMEGA*A/G
0
0.6
1.2
1.8
2.4
3
0
1.2
2.4
3.6
4.8
6

Figure 17

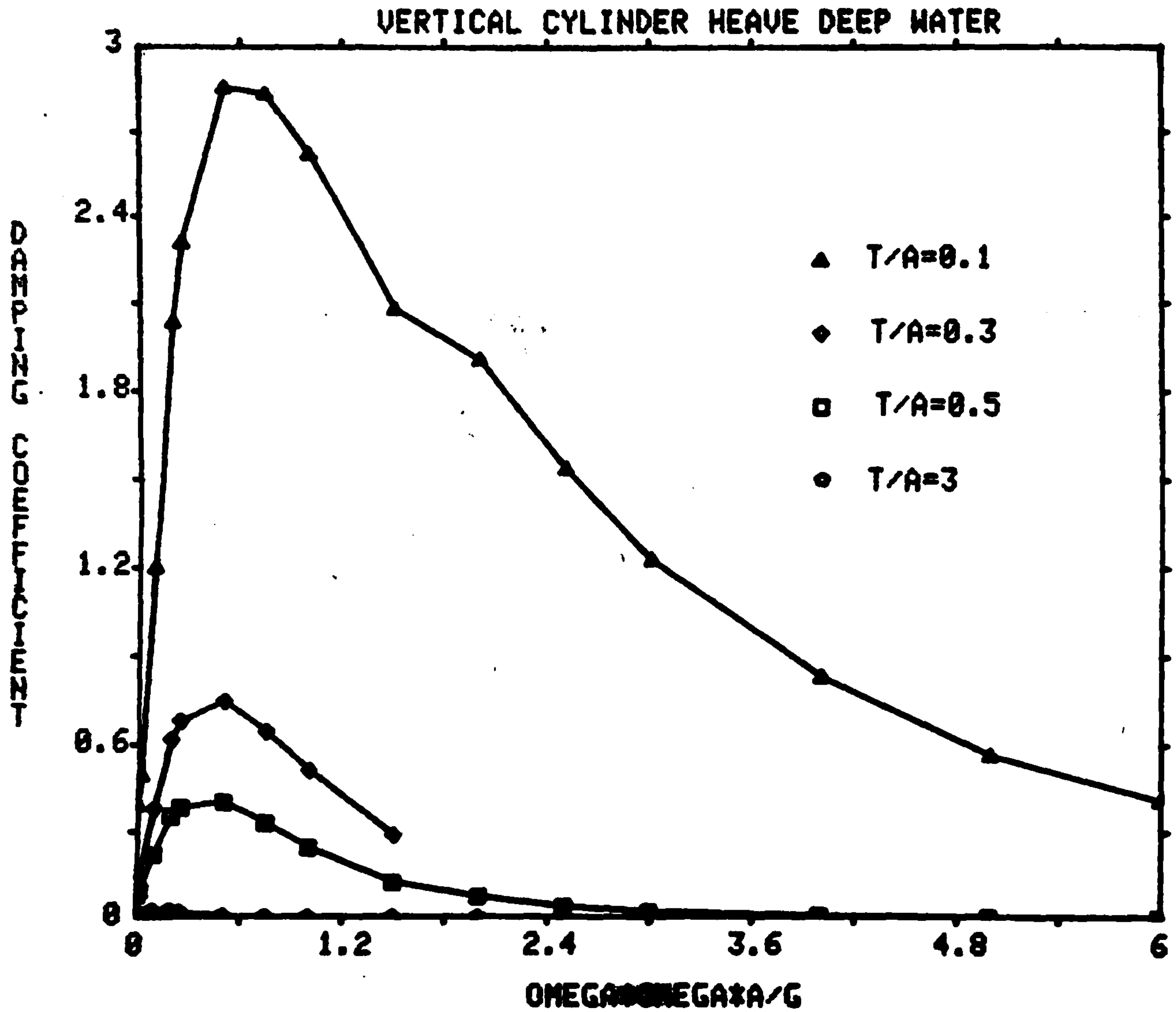
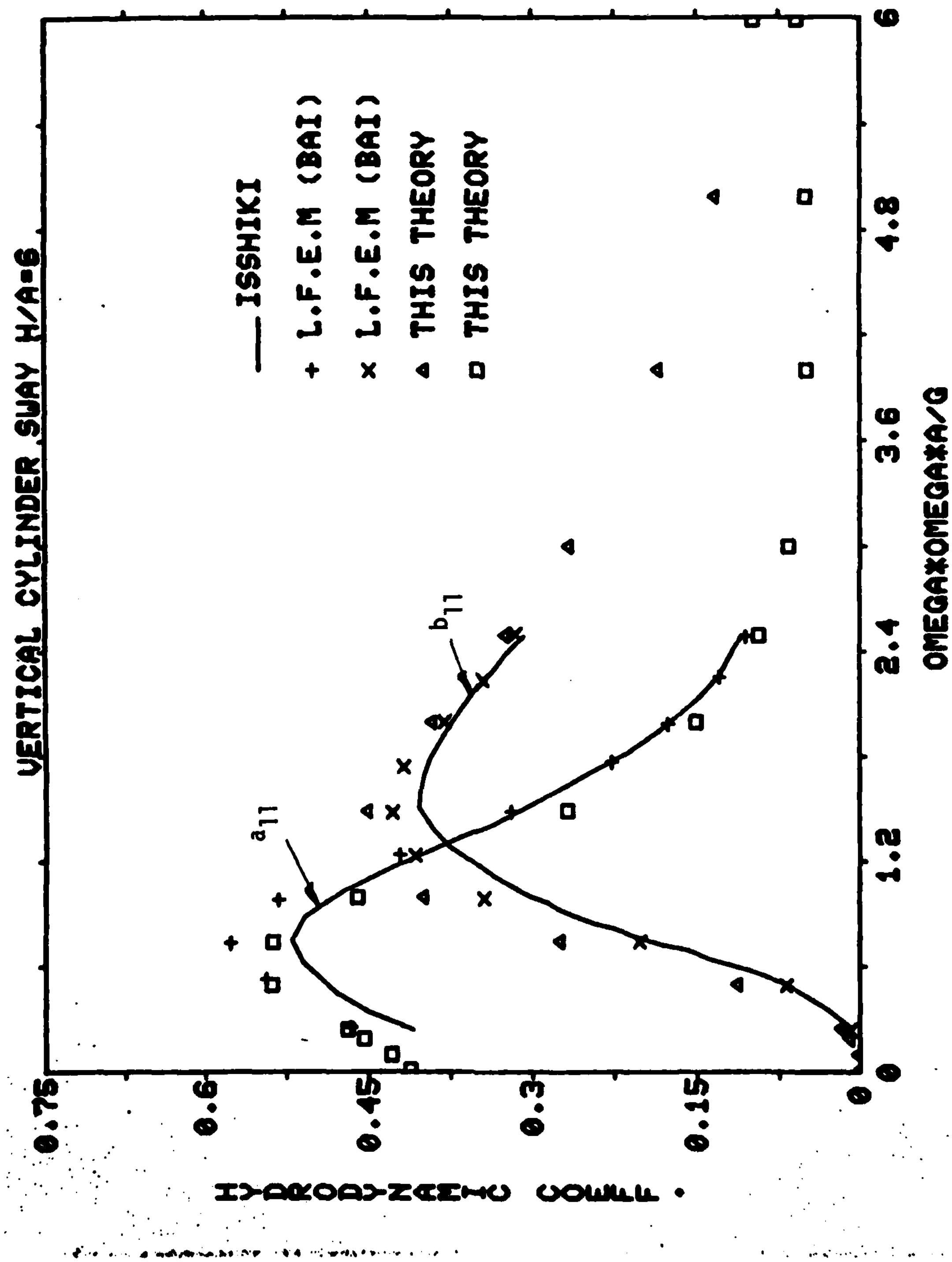

VERTICAL CYLINDER HEAVE DEEP WATER
DAMPING COEFFICIENT
3
2.4
1.8
1.2
0.6
0
0
1.2
2.4
3.6
4.8
6
T/A=0.1
T/A=0.3
T/A=0.5
T/A=3
OMEGA*OMEGA*A/G

Figure 18



VERTICAL CYLINDER SWAY H/A=6
ISSHIKI
+ L.F.E.M (BAI)
× L.F.E.M (BAI)
THIS THEORY
THIS THEORY
a_{11}
b_{11}
HYDRODYNAMIC COEFF.
0.75
0.6
0.45
0.3
0.15
0
1.2
2.4
3.6
4.8
6
OMEGAXOMEGAXA/G

Figure 19

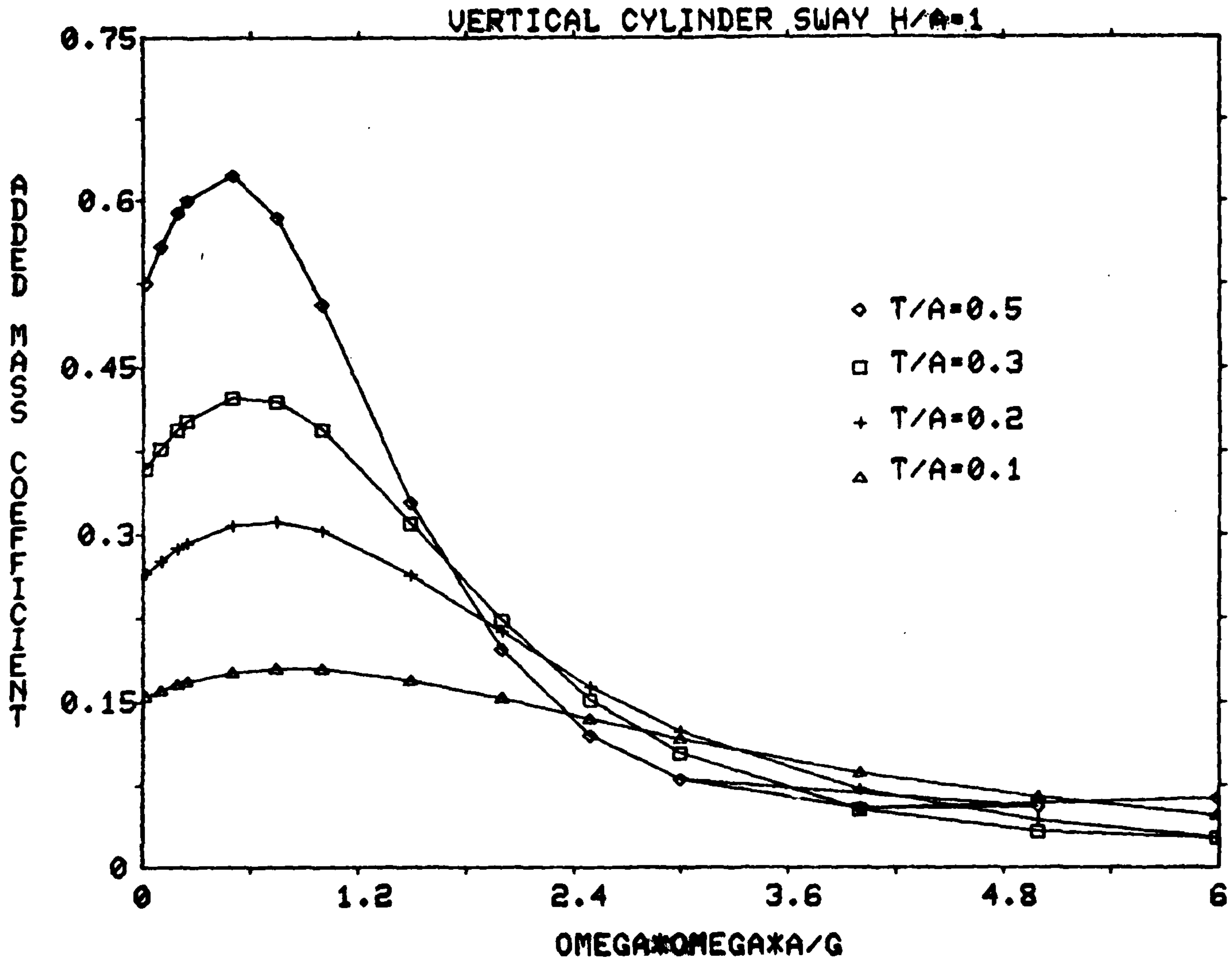

VERTICAL CYLINDER SWAY H/A=1
ADDED MASS COEFFICIENT
0.75
0.6
0.45
0.3
0.15
0
1.2
2.4
3.6
4.8
6
OMEGA*OMEGA*A/G
T/A=0.5
T/A=0.3
T/A=0.2
T/A=0.1

Figure 20

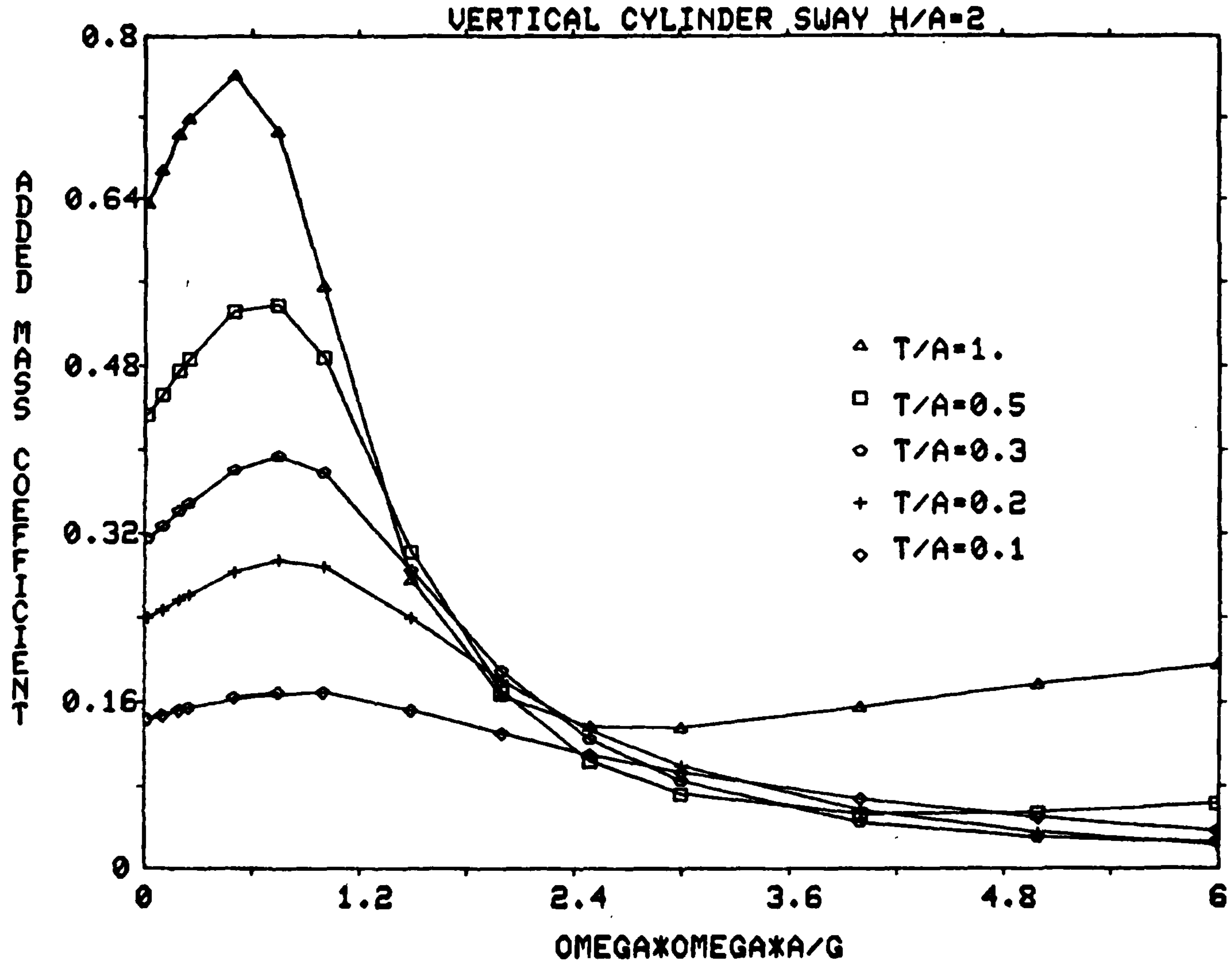

VERTICAL CYLINDER SWAY H/A=2
T/A=1.
T/A=0.5
T/A=0.3
T/A=0.2
T/A=0.1
ADDED MASS COEFFICIENT
OMEGA*OMEGA*A/G
0
0.16
0.32
0.48
0.64
0.8
1.2
2.4
3.6
4.8
6

Figure 21

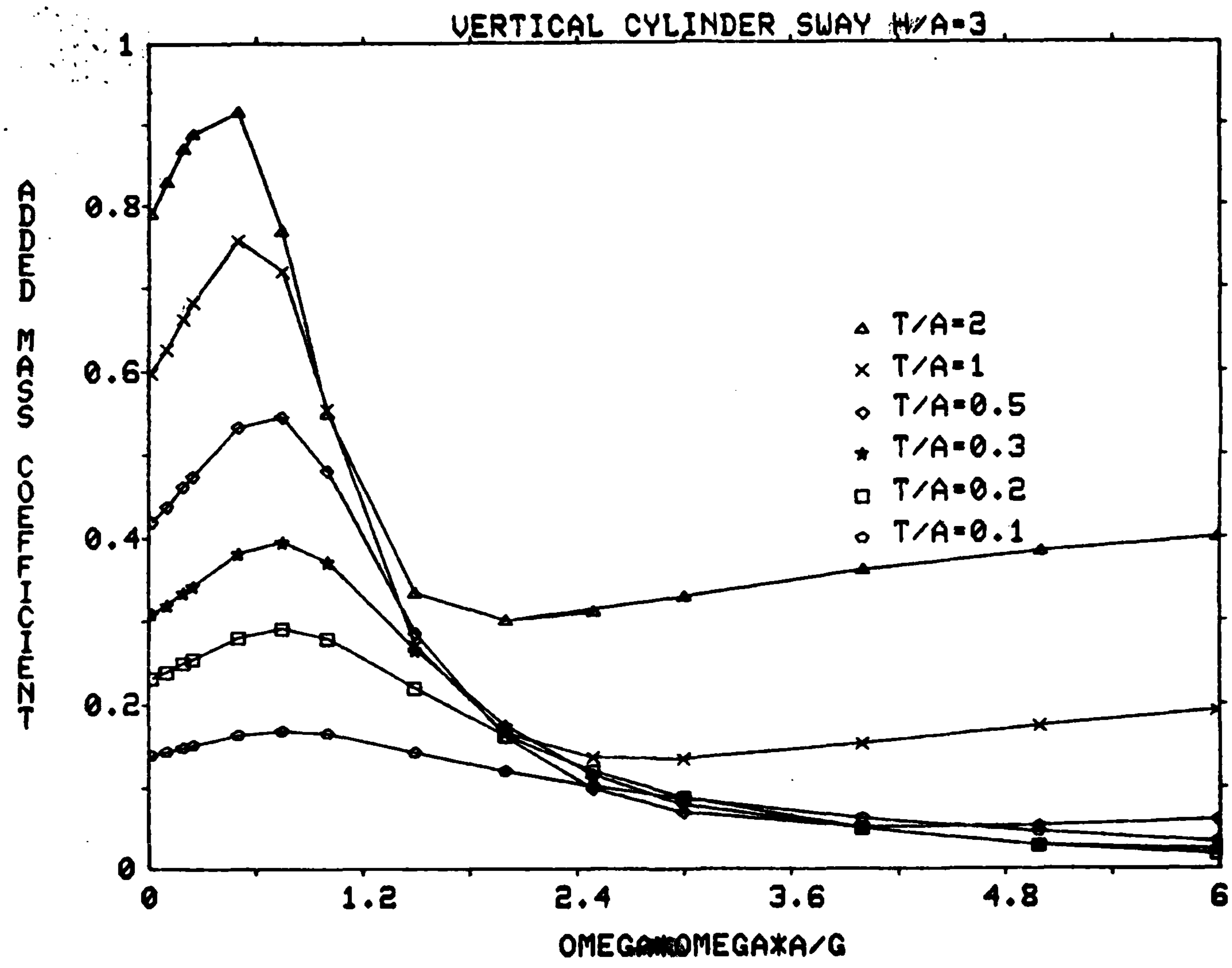

VERTICAL CYLINDER SWAY H/A=3
ADDED MASS COEFFICIENT
OMEGA*OMEGA*A/G
T/A=2
T/A=1
T/A=0.5
T/A=0.3
T/A=0.2
T/A=0.1
0
0.2
0.4
0.6
0.8
1
1.2
2.4
3.6
4.8
6

Figure 22

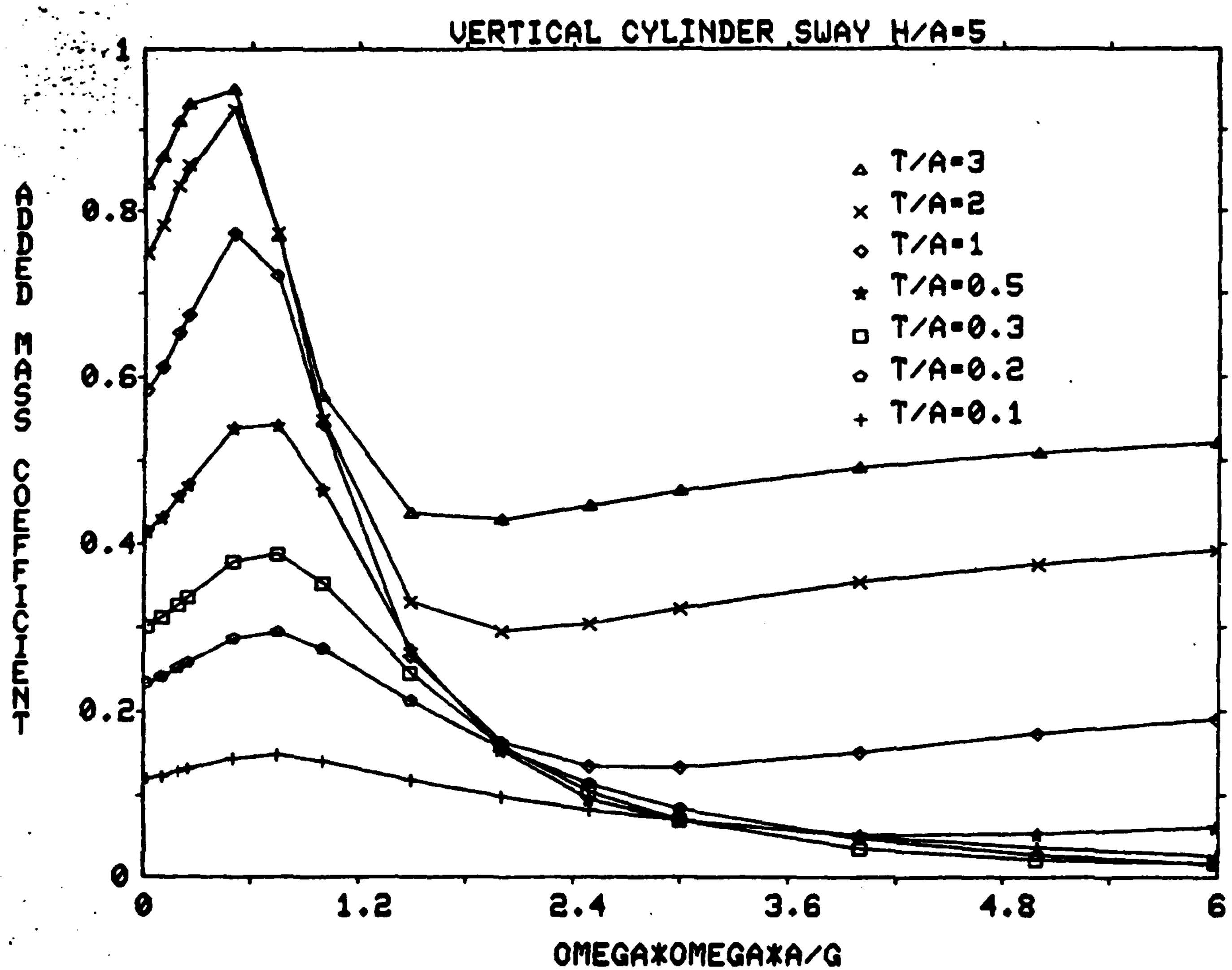
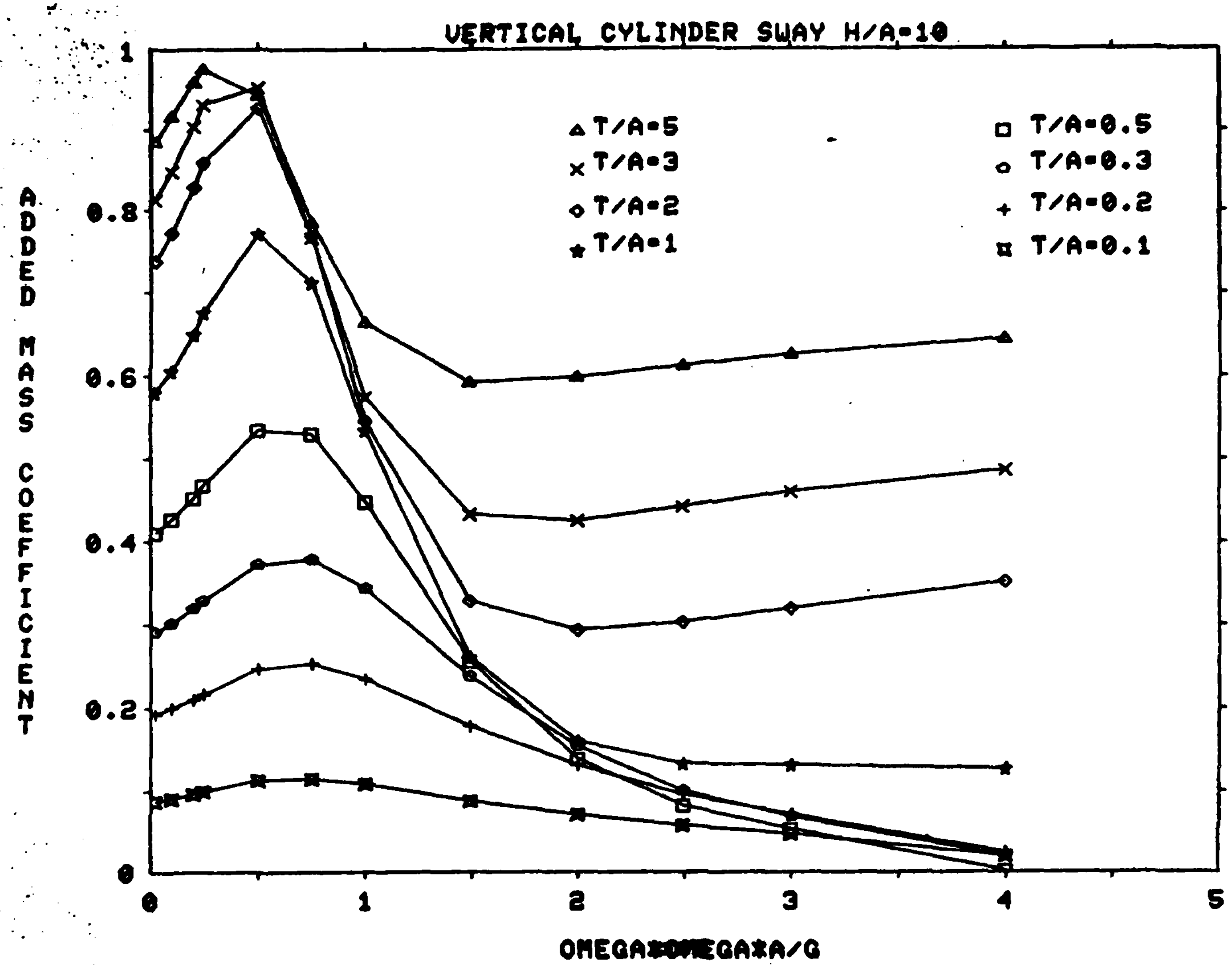

VERTICAL CYLINDER SWAY H/A=5
ADDED MASS COEFFICIENT
OMEGA*OMEGA*A/G
T/A=3
T/A=2
T/A=1
T/A=0.5
T/A=0.3
T/A=0.2
T/A=0.1
0
0.2
0.4
0.6
0.8
1
1.2
2.4
3.6
4.8
6

Figure 23



VERTICAL CYLINDER SWAY H/A=10
T/A=5
T/A=3
T/A=2
T/A=1
T/A=0.5
T/A=0.3
T/A=0.2
T/A=0.1
ADDED MASS COEFFICIENT
OMEGA*OMEGA*A/G
0
0.2
0.4
0.6
0.8
1
0
1
2
3
4
5

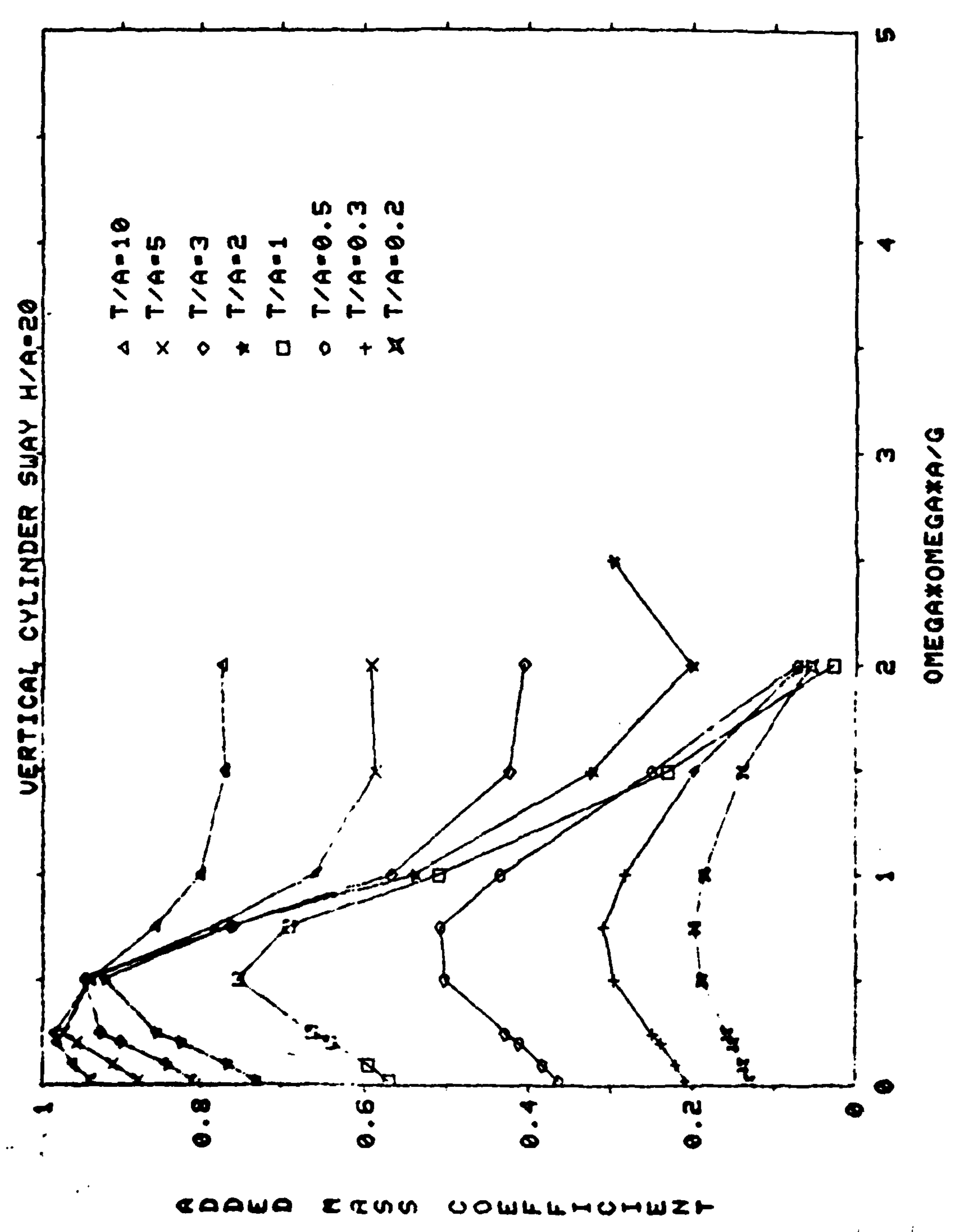
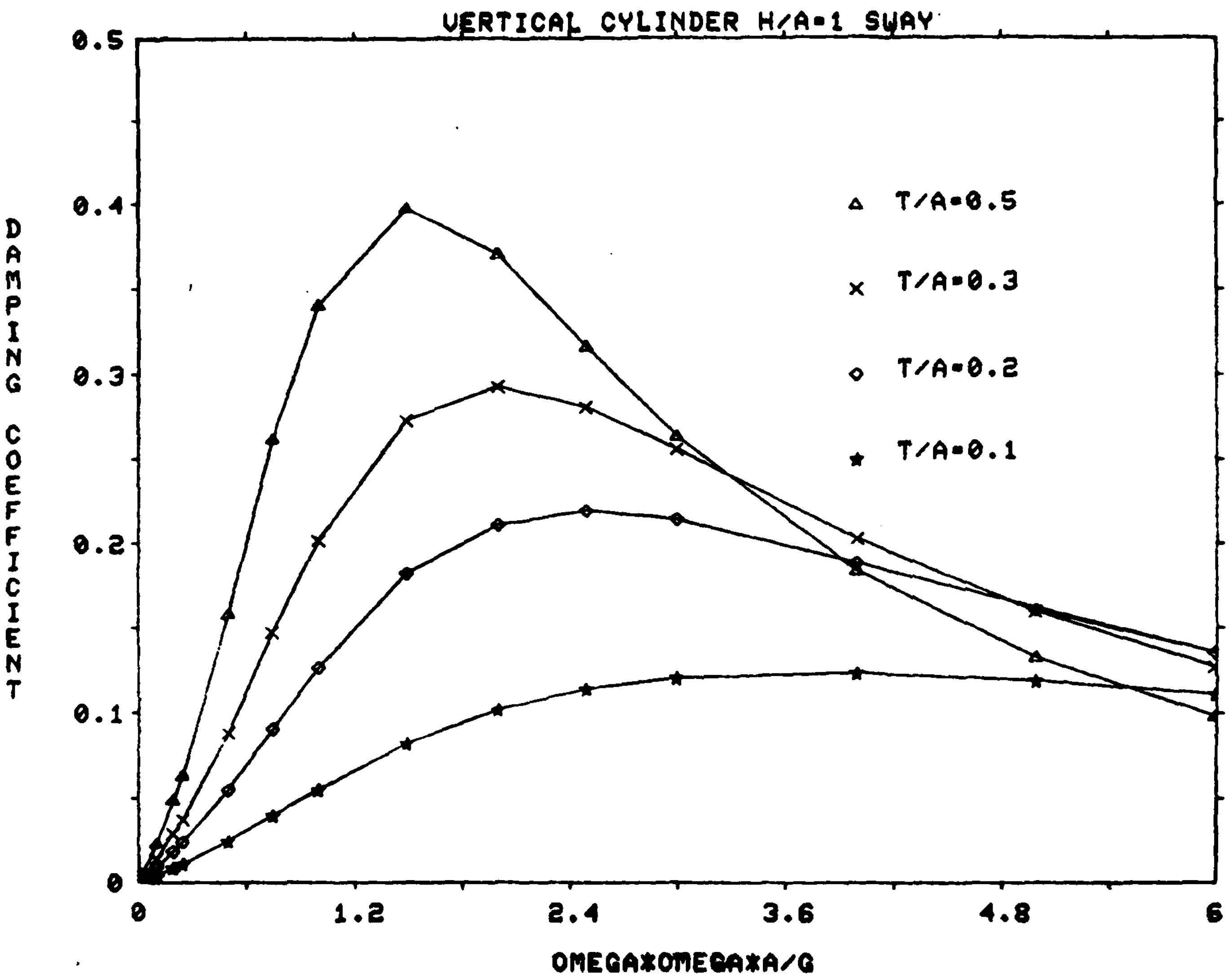

VERTICAL CYLINDER SWAY H/A=20
T/A=10
T/A=5
T/A=3
T/A=2
T/A=1
T/A=0.5
T/A=0.3
T/A=0.2
OMEGA*OMEGA*A/G
ADDED MASS COEFFICIENT
0
1
2
3
4
5
0.2
0.4
0.6
0.8
1

Figure 25



VERTICAL CYLINDER H/A=1 SWAY
T/A=0.5
T/A=0.3
T/A=0.2
T/A=0.1
DAMPING COEFFICIENT
OMEGA*OMEGA*A/G
0
0.1
0.2
0.3
0.4
0.5
1.2
2.4
3.6
4.8
6

Figure 26

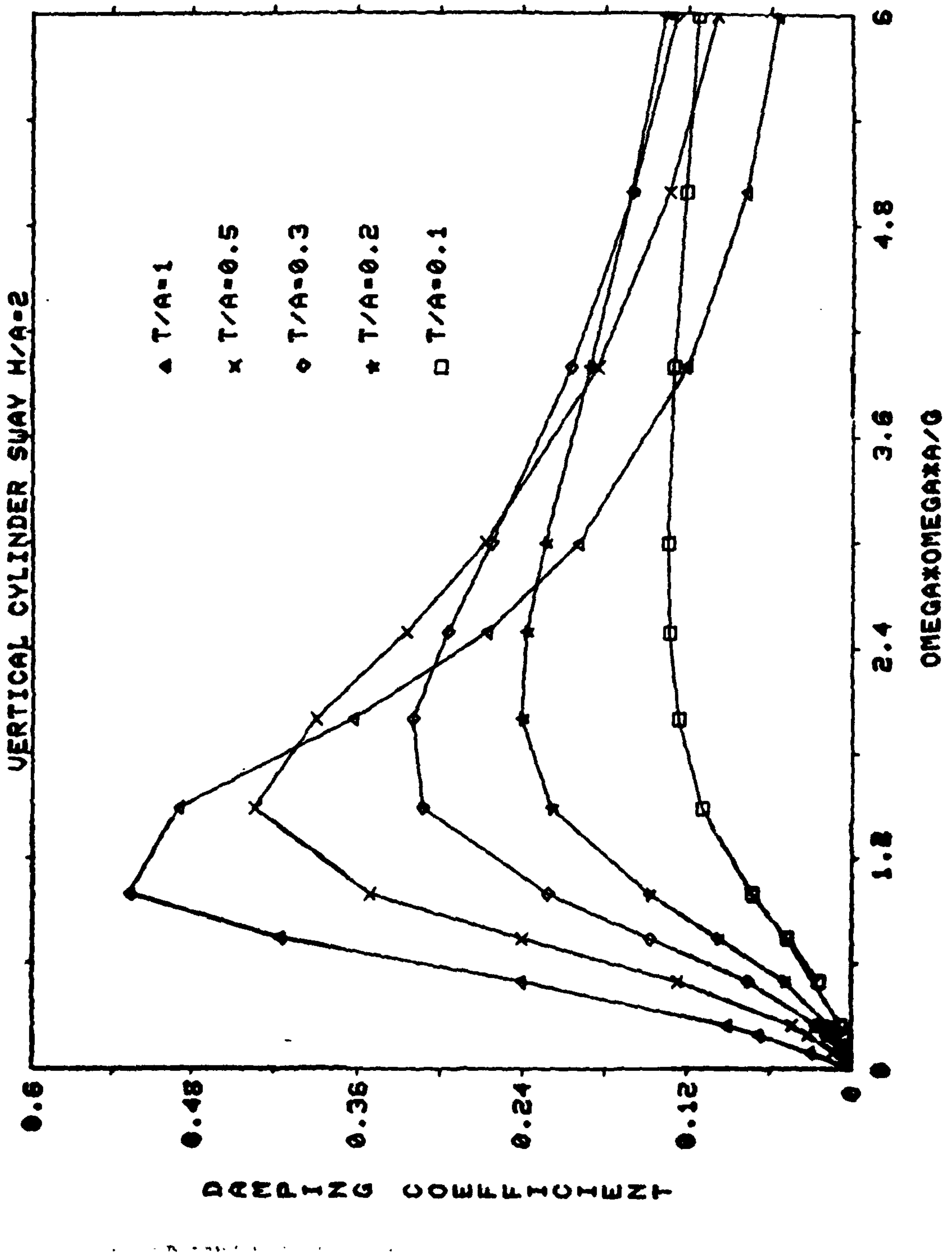

VERTICAL CYLINDER SWAY H/A=2
T/A=1
T/A=0.5
T/A=0.3
T/A=0.2
T/A=0.1
DAMPING COEFFICIENT
0.6
0.48
0.36
0.24
0.12
0
1.2
2.4
3.6
4.8
6
OMEGAXOMEGAXA/G

Figure 27

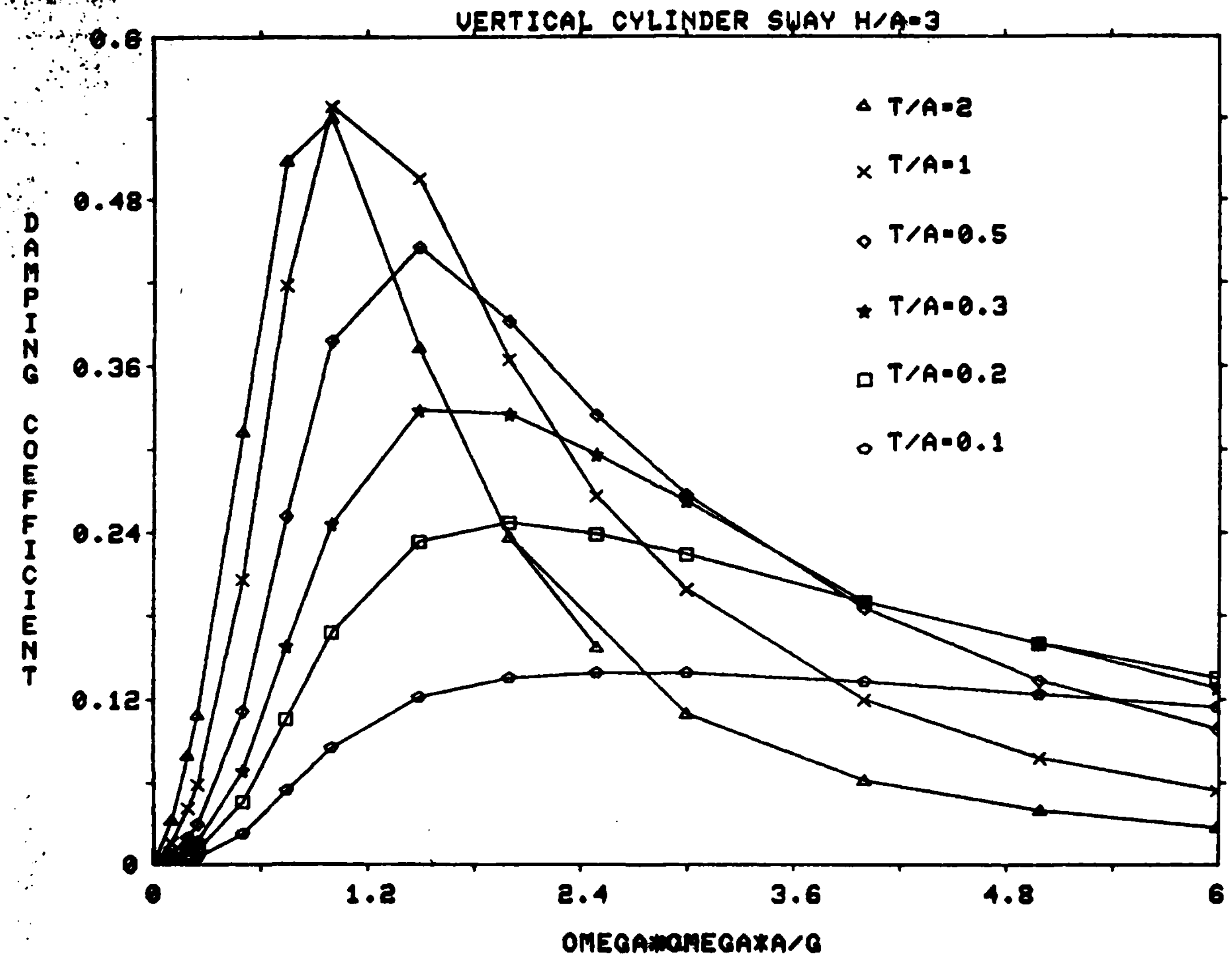

VERTICAL CYLINDER SWAY H/A=3
DAMPING COEFFICIENT
0.6
0.48
0.36
0.24
0.12
0
1.2
2.4
3.6
4.8
6
OMEGA*OMEGA*A/G
T/A=2
T/A=1
T/A=0.5
T/A=0.3
T/A=0.2
T/A=0.1

Figure 28

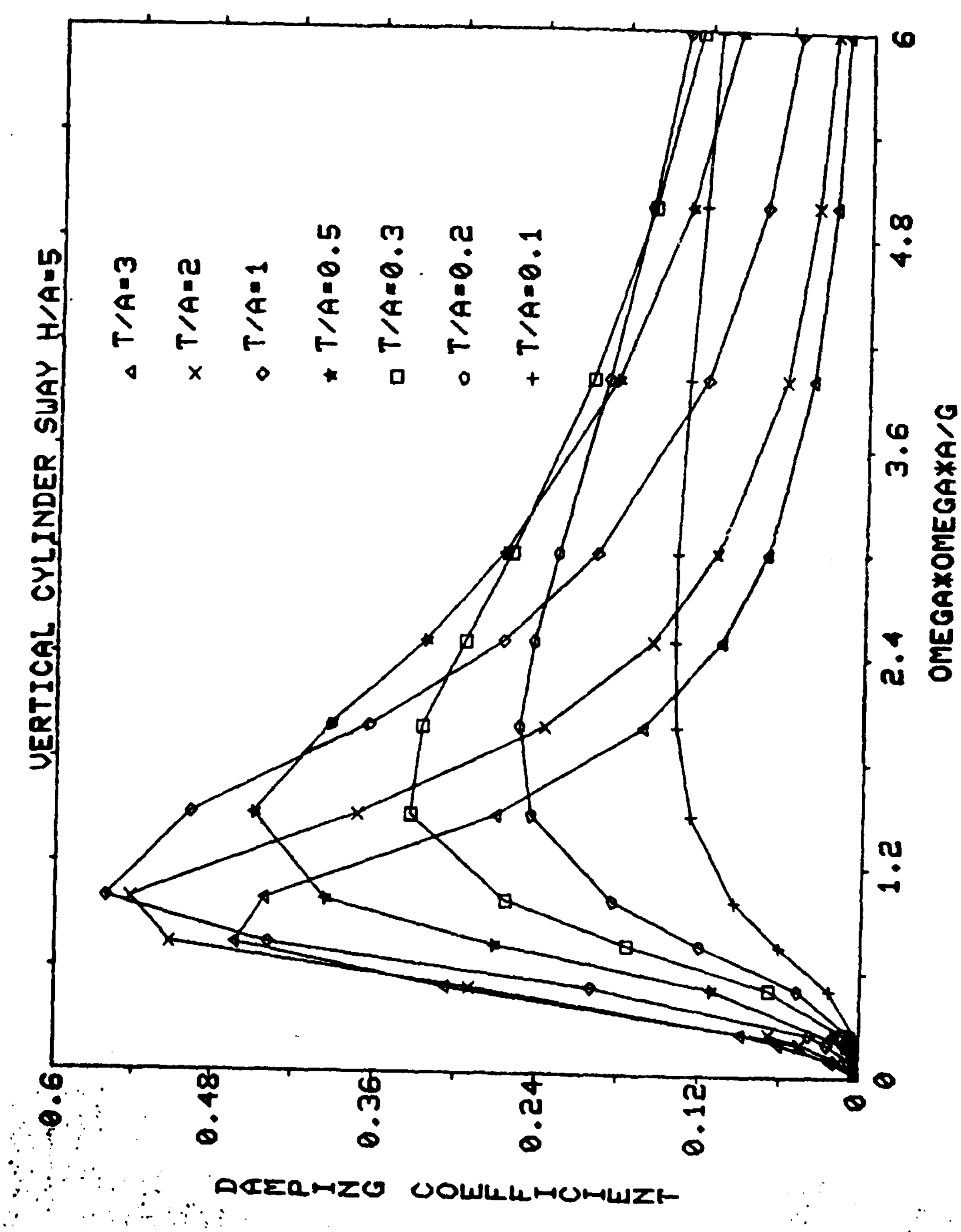

VERTICAL CYLINDER SWAY H/A=5
T/A=3
T/A=2
T/A=1
T/A=0.5
T/A=0.3
T/A=0.2
T/A=0.1
0.6
0.48
0.36
0.24
0.12
0
1.2
2.4
3.6
4.8
6
OMEGA*OMEGA*A/G
DAMPING COEFFICIENT

Figure 29

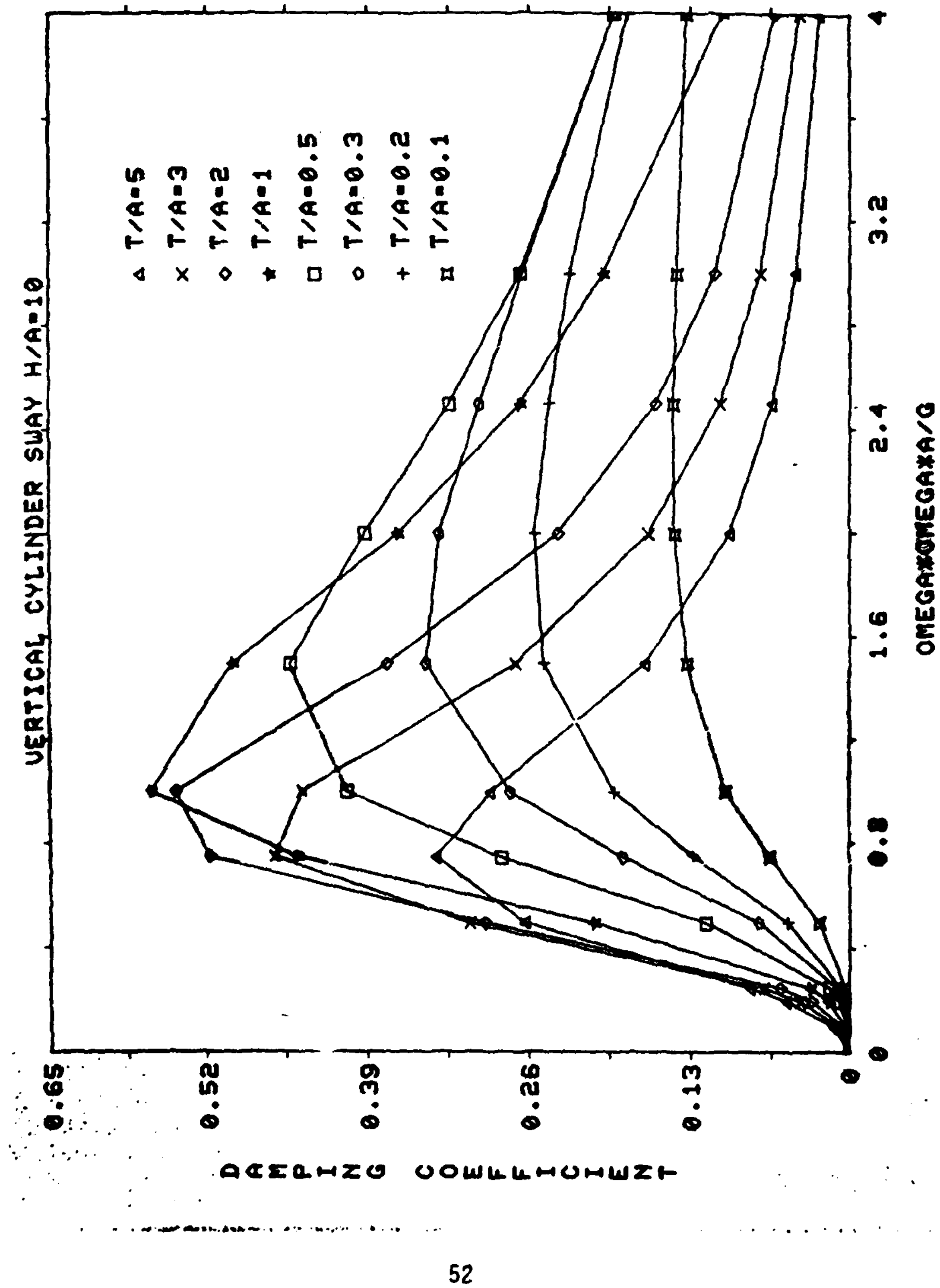

VERTICAL CYLINDER SWAY H/A=10
T/A=5
T/A=3
T/A=2
T/A=1
T/A=0.5
T/A=0.3
T/A=0.2
T/A=0.1
DAMPING COEFFICIENT
0.65
0.52
0.39
0.26
0.13
0
0.8
1.6
2.4
3.2
4
OMEGA*OMEGA*A/G

Figure 30

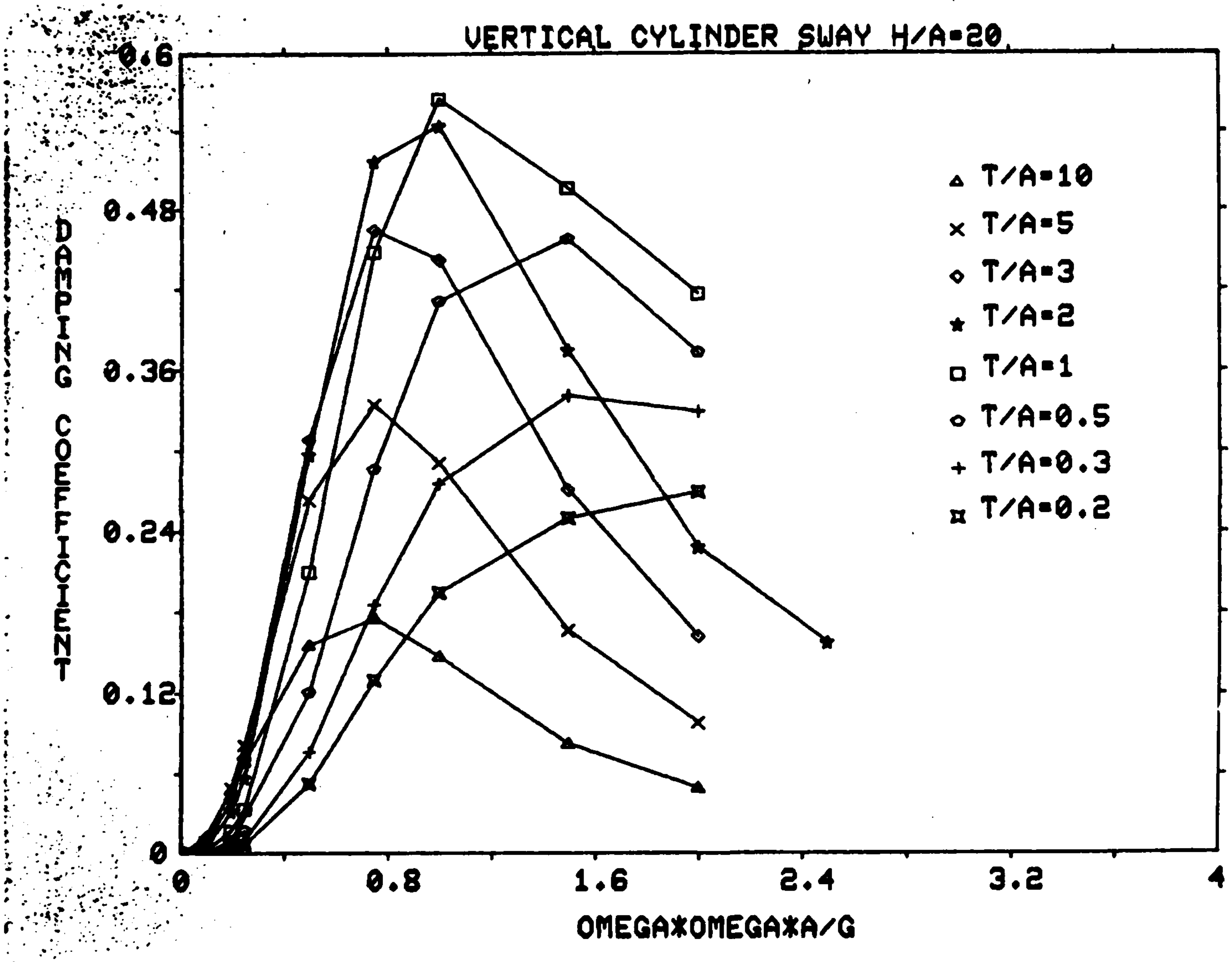
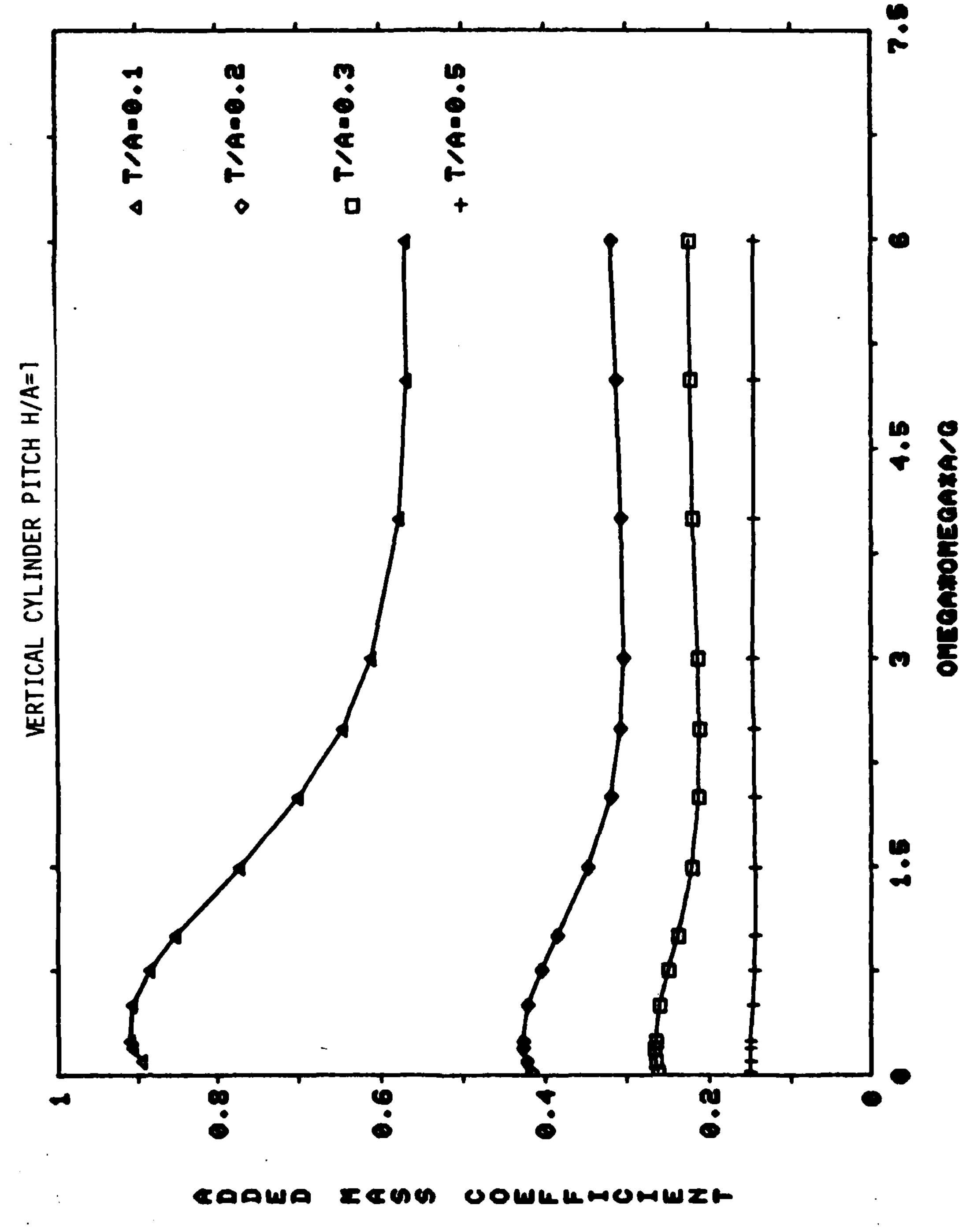

VERTICAL CYLINDER SWAY H/A=20
DAMPING COEFFICIENT
0.6
0.48
0.36
0.24
0.12
0
0.8
1.6
2.4
3.2
4
OMEGA*OMEGA*A/G
T/A=10
T/A=5
T/A=3
T/A=2
T/A=1
T/A=0.5
T/A=0.3
T/A=0.2

Figure 31



VERTICAL CYLINDER PITCH H/A=1
T/A=0.1
T/A=0.2
T/A=0.3
T/A=0.5
OMEGA*OMEGA*A/G
0
1.5
3
4.5
6
7.5
0
0.2
0.4
0.6
0.8
1

Figure 32

VERTICAL CYLINDER PITCH H/A=2

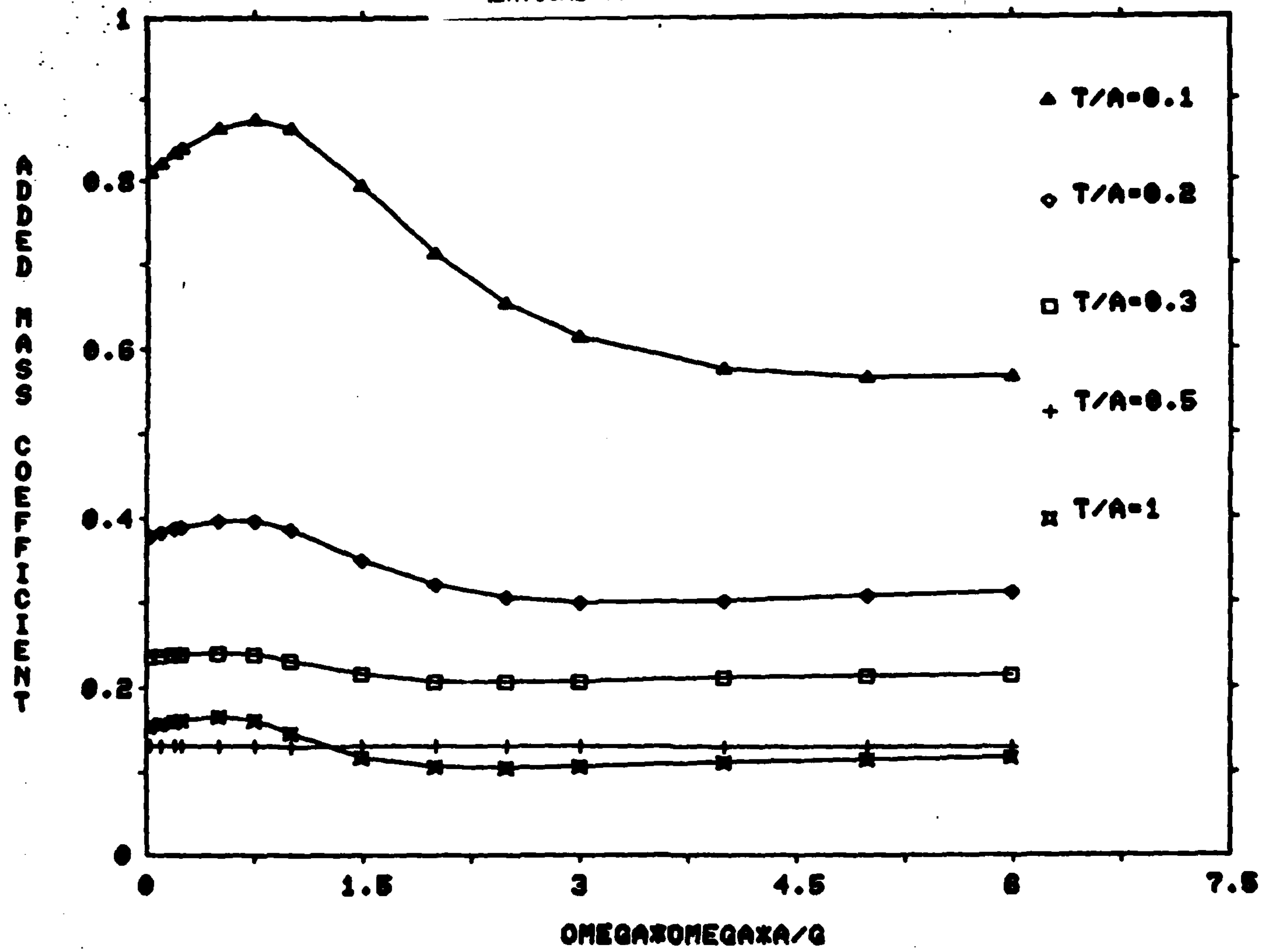


Figure 33

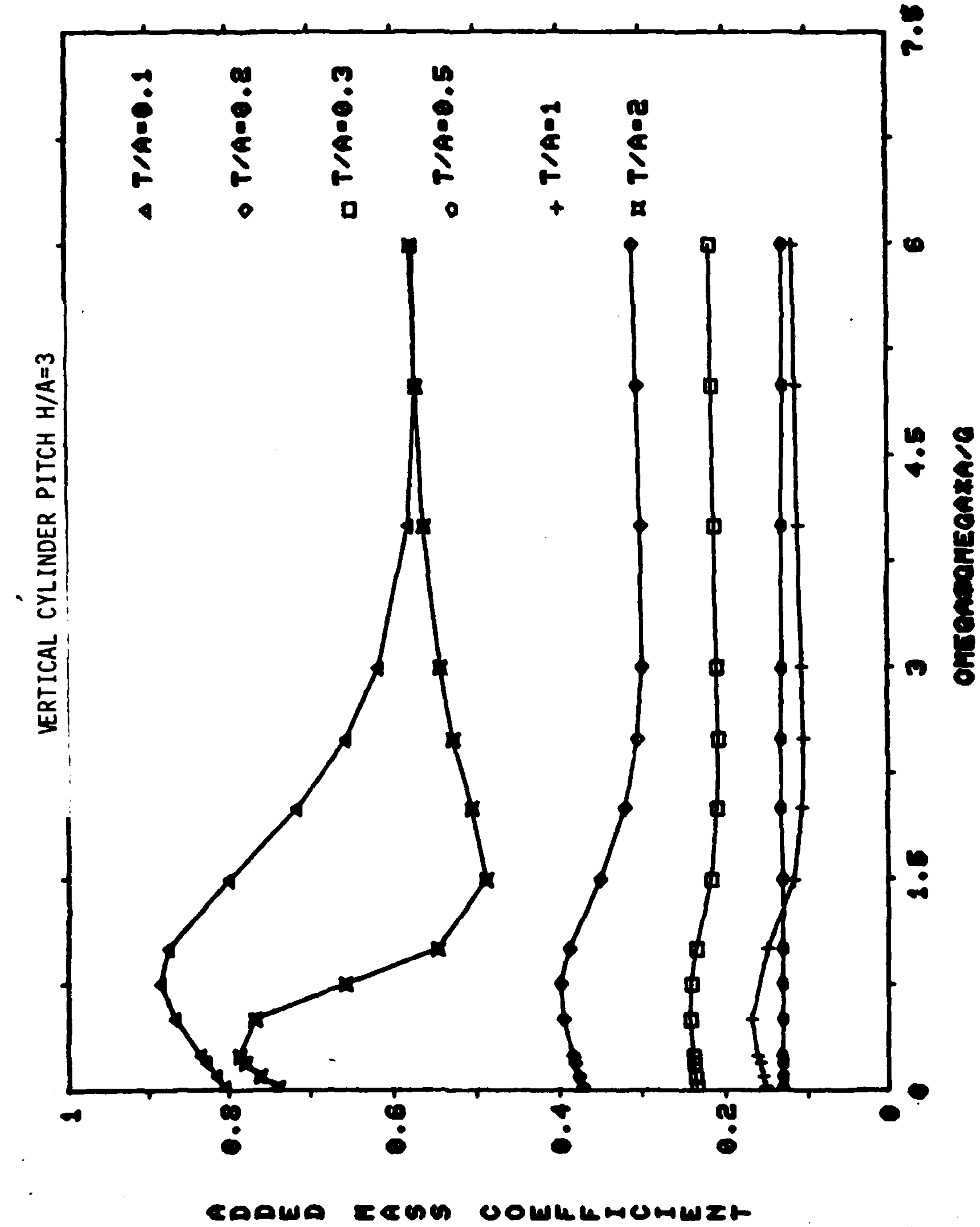

VERTICAL CYLINDER PITCH H/A=3
T/A=0.1
T/A=0.2
T/A=0.3
T/A=0.5
T/A=1
T/A=2
0
0.2
0.4
0.6
0.8
1
0
1.5
3
4.5
6
7.5

Figure 34

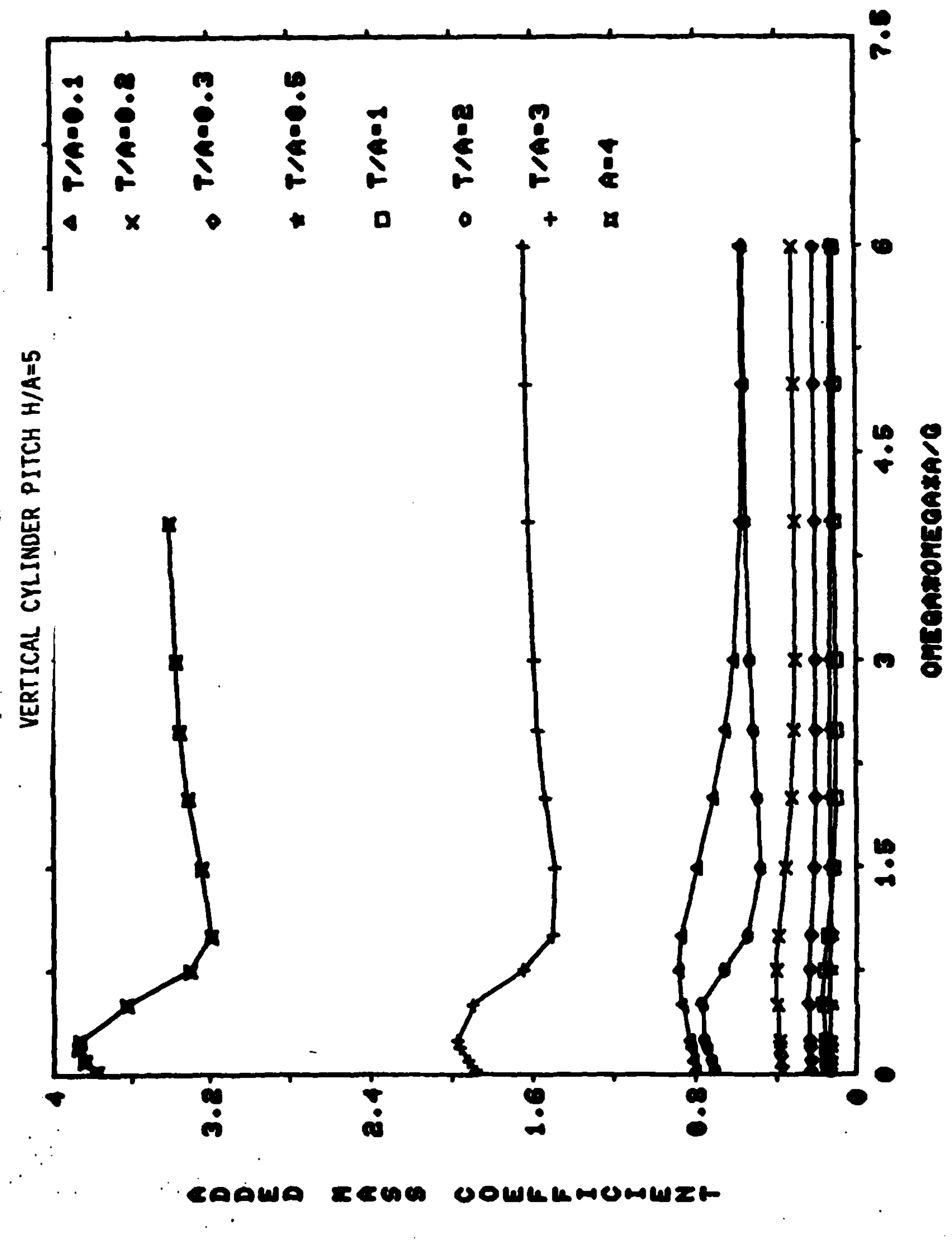

VERTICAL CYLINDER PITCH H/A=5
T/A=0.1
T/A=0.2
T/A=0.3
T/A=0.5
T/A=1
T/A=2
T/A=3
A=4
0
0.8
1.6
2.4
3.2
4
0
1.5
3
4.5
6
7.5
OMEGA*OMEGA*A/G

Figure 35

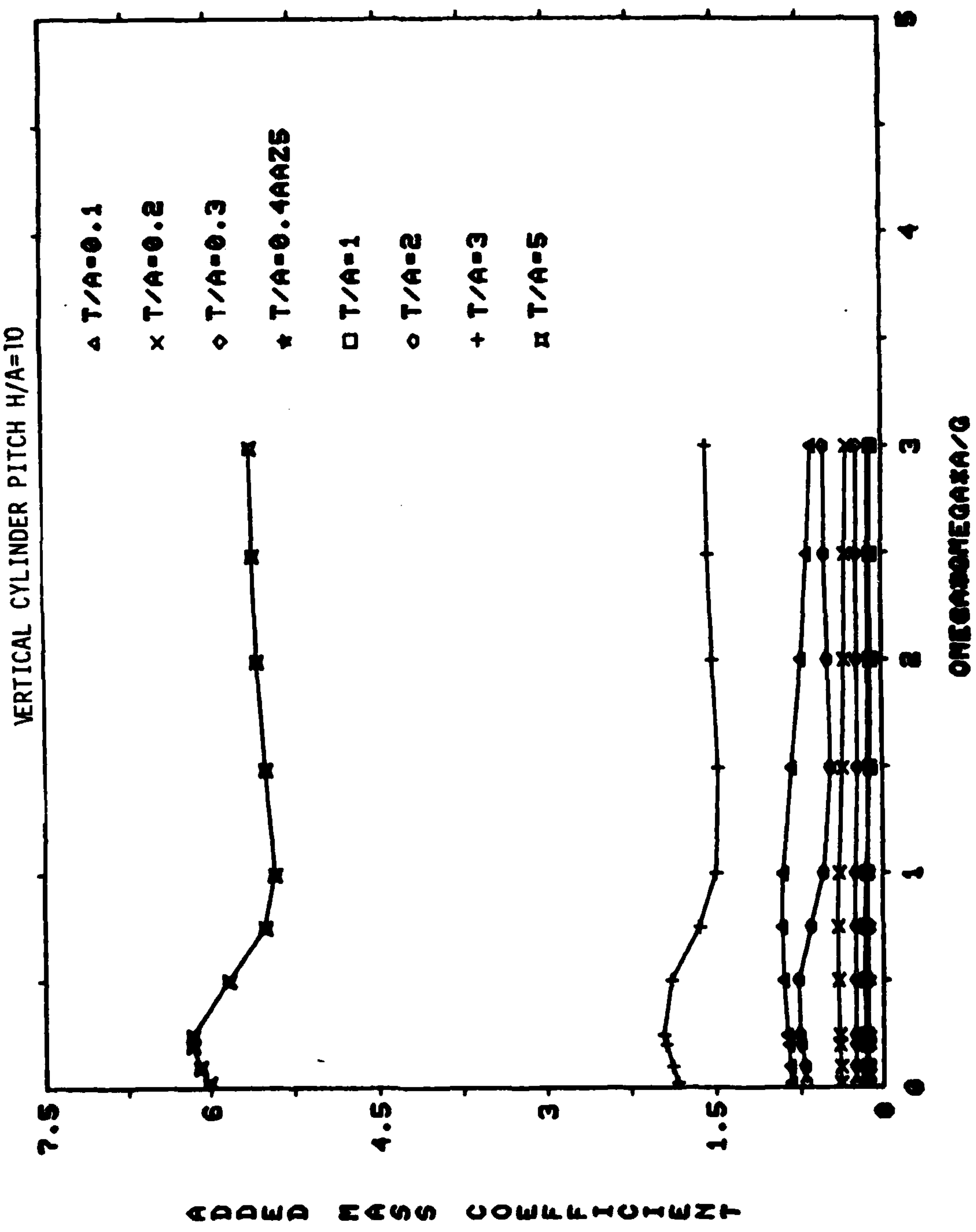

VERTICAL CYLINDER PITCH H/A=10
T/A=0.1
T/A=0.2
T/A=0.3
T/A=0.4AAZ5
T/A=1
T/A=2
T/A=3
T/A=5
ADDED MASS COEFFICIENT
OMEGA*OMEGA*A/G
7.5
6
4.5
3
1.5
0
1
2
3
4
5

Figure 36

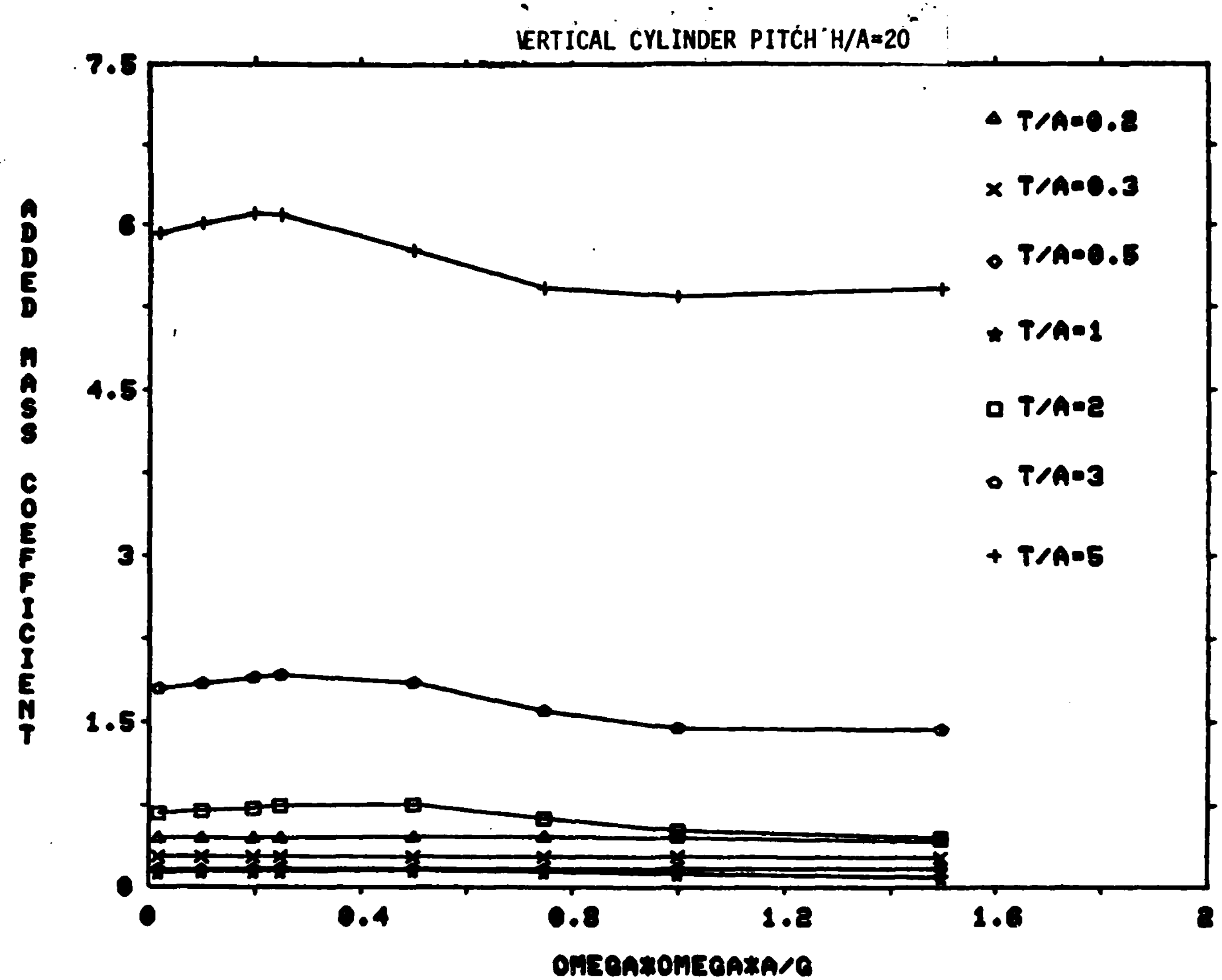

VERTICAL CYLINDER PITCH H/A=20
ADDED MASS COEFFICIENT
7.5
6
4.5
3
1.5
0
0.4
0.8
1.2
1.6
2
OMEGA*OMEGA*A/G
T/A=0.2
T/A=0.3
T/A=0.5
T/A=1
T/A=2
T/A=3
T/A=5

Figure 37

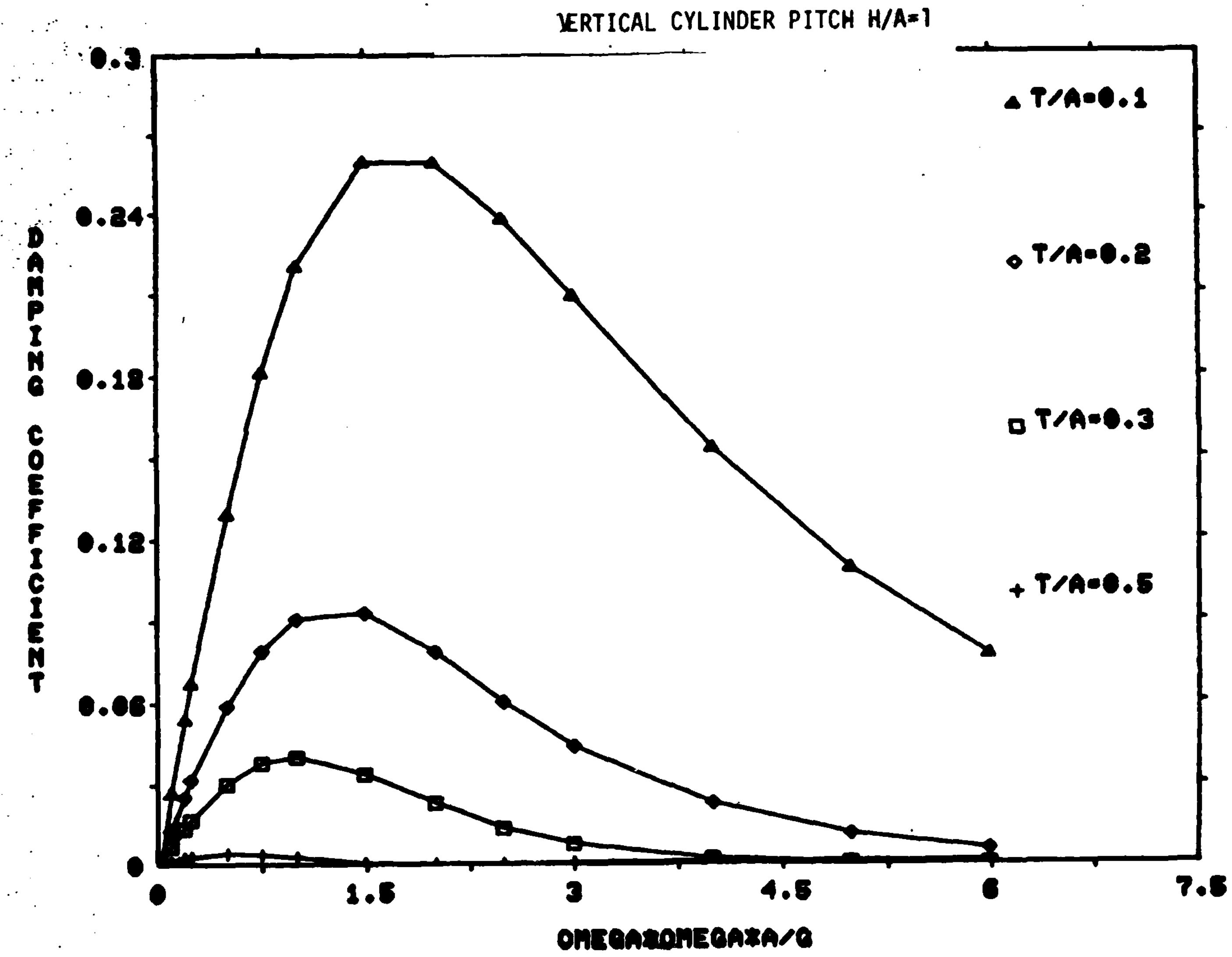

VERTICAL CYLINDER PITCH H/A=1
DAMPING COEFFICIENT
OMEGA*OMEGA*A/G
T/A=0.1
T/A=0.2
T/A=0.3
T/A=0.5
0
0.06
0.12
0.18
0.24
0.3
1.5
3
4.5
6
7.5

Figure 38

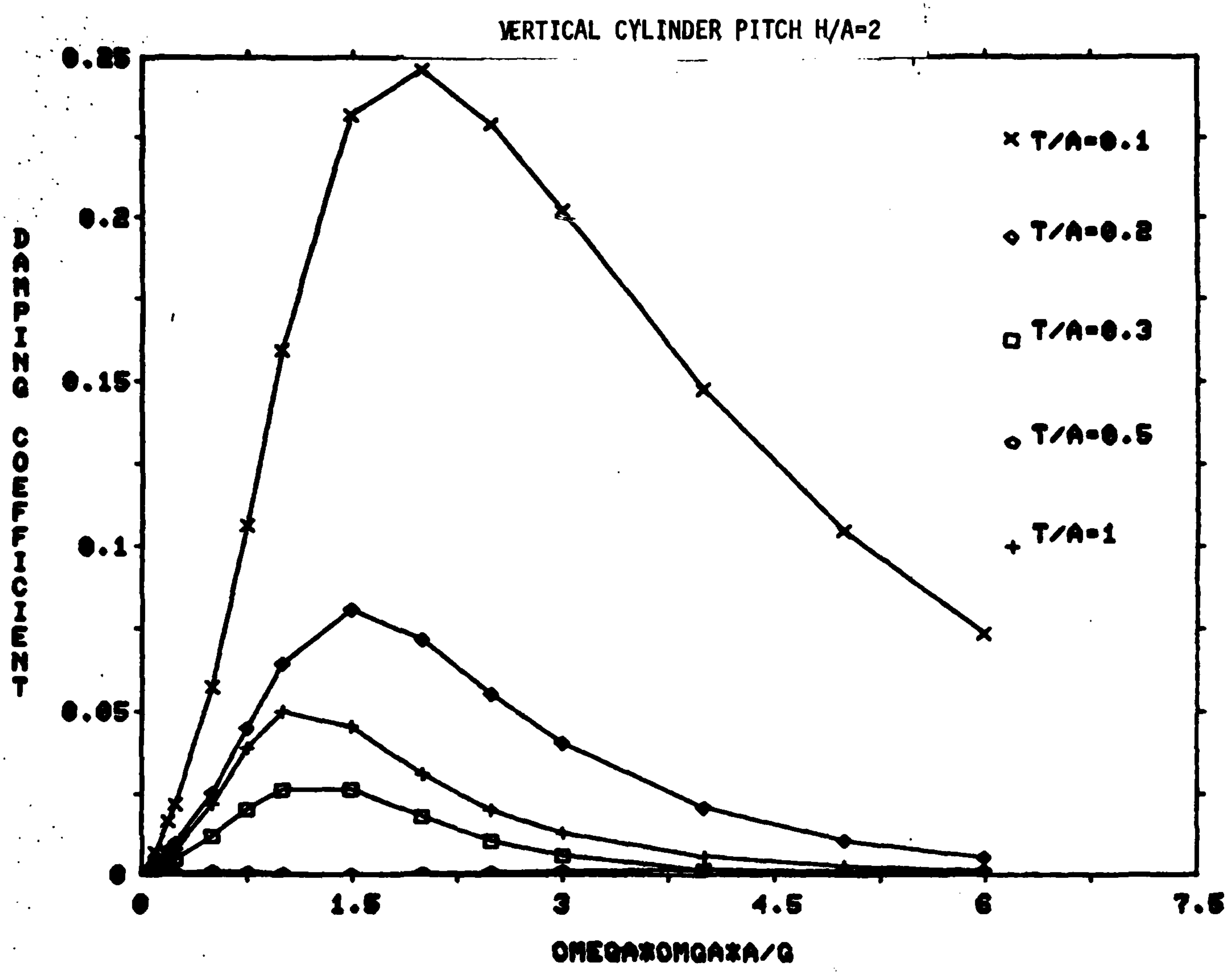

VERTICAL CYLINDER PITCH H/A=2
DAMPING COEFFICIENT
OMEGA*OMGA*A/G
0
0.05
0.1
0.15
0.2
0.25
1.5
3
4.5
6
7.5
T/A=0.1
T/A=0.2
T/A=0.3
T/A=0.5
T/A=1

Figure 39

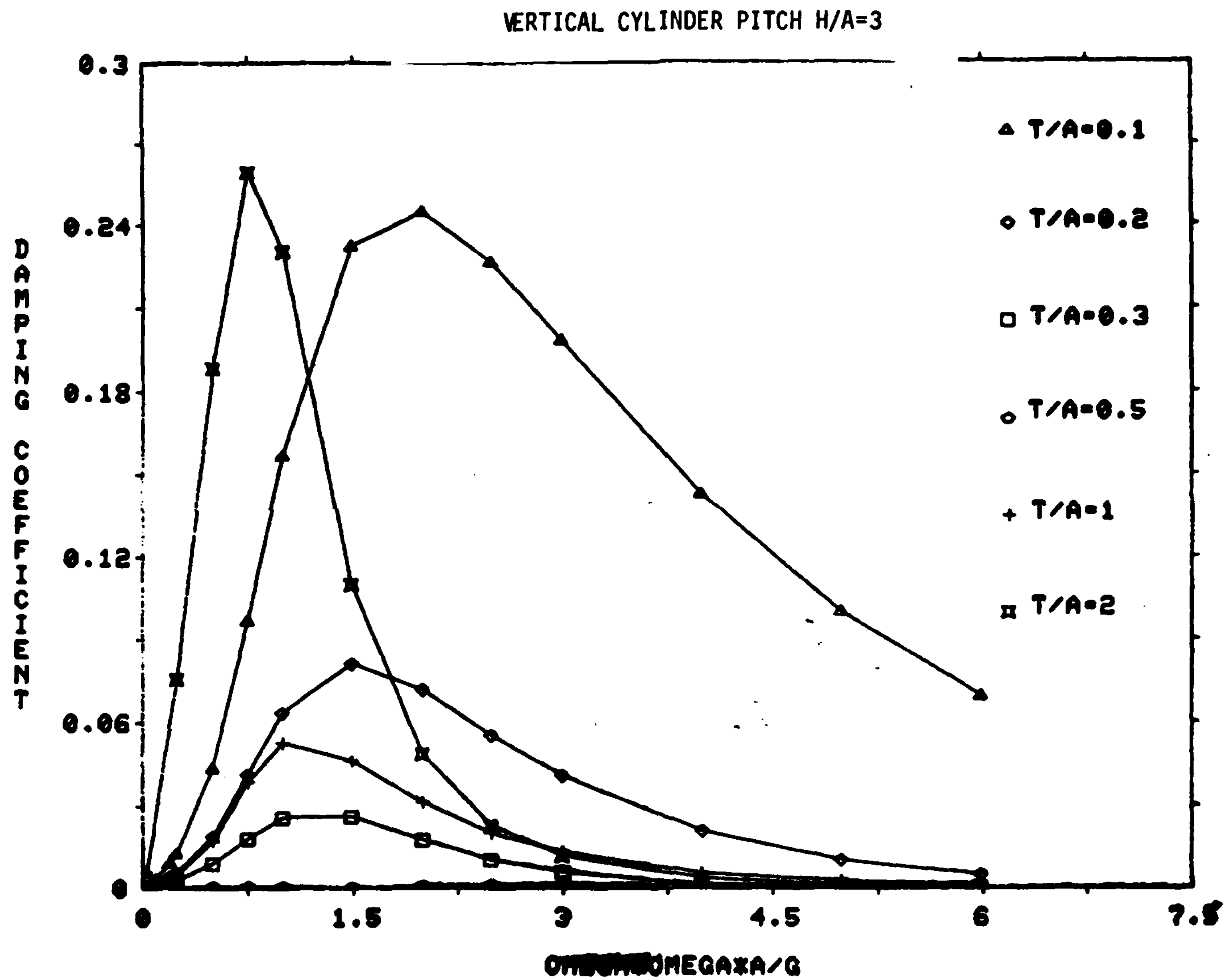

VERTICAL CYLINDER PITCH H/A=3
DAMPING COEFFICIENT
0.3
0.24
0.18
0.12
0.06
0
1.5
3
4.5
6
7.5
OMEGA*A/G
T/A=0.1
T/A=0.2
T/A=0.3
T/A=0.5
T/A=1
T/A=2

Figure 40

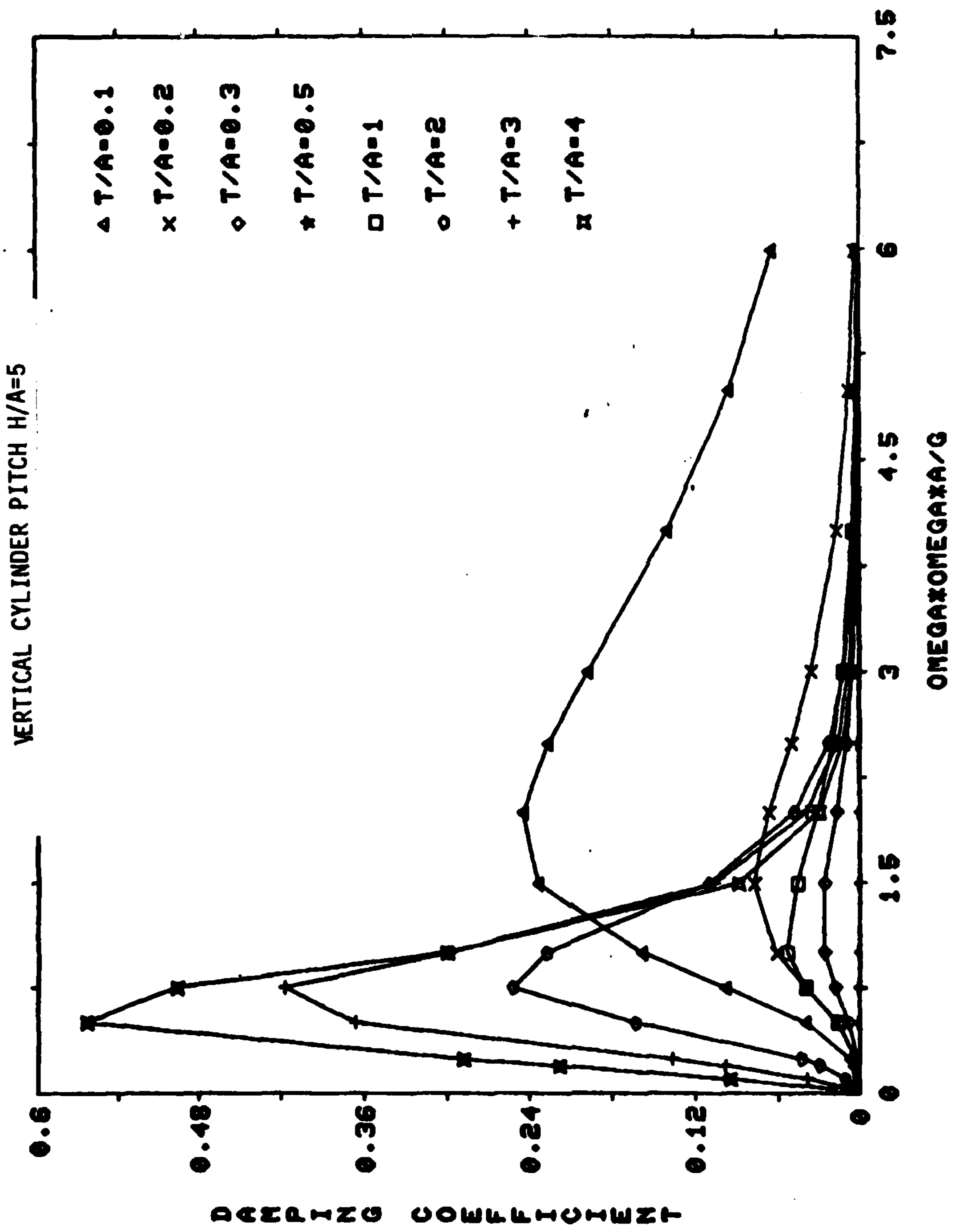

VERTICAL CYLINDER PITCH H/A=5
T/A=0.1
T/A=0.2
T/A=0.3
T/A=0.5
T/A=1
T/A=2
T/A=3
T/A=4
DAMPING COEFFICIENT
0.6
0.48
0.36
0.24
0.12
0
1.5
3
4.5
6
7.5
OMEGA*OMEGA*A/G

Figure 41

VERTICAL CYLINDER PITCH H/A=10

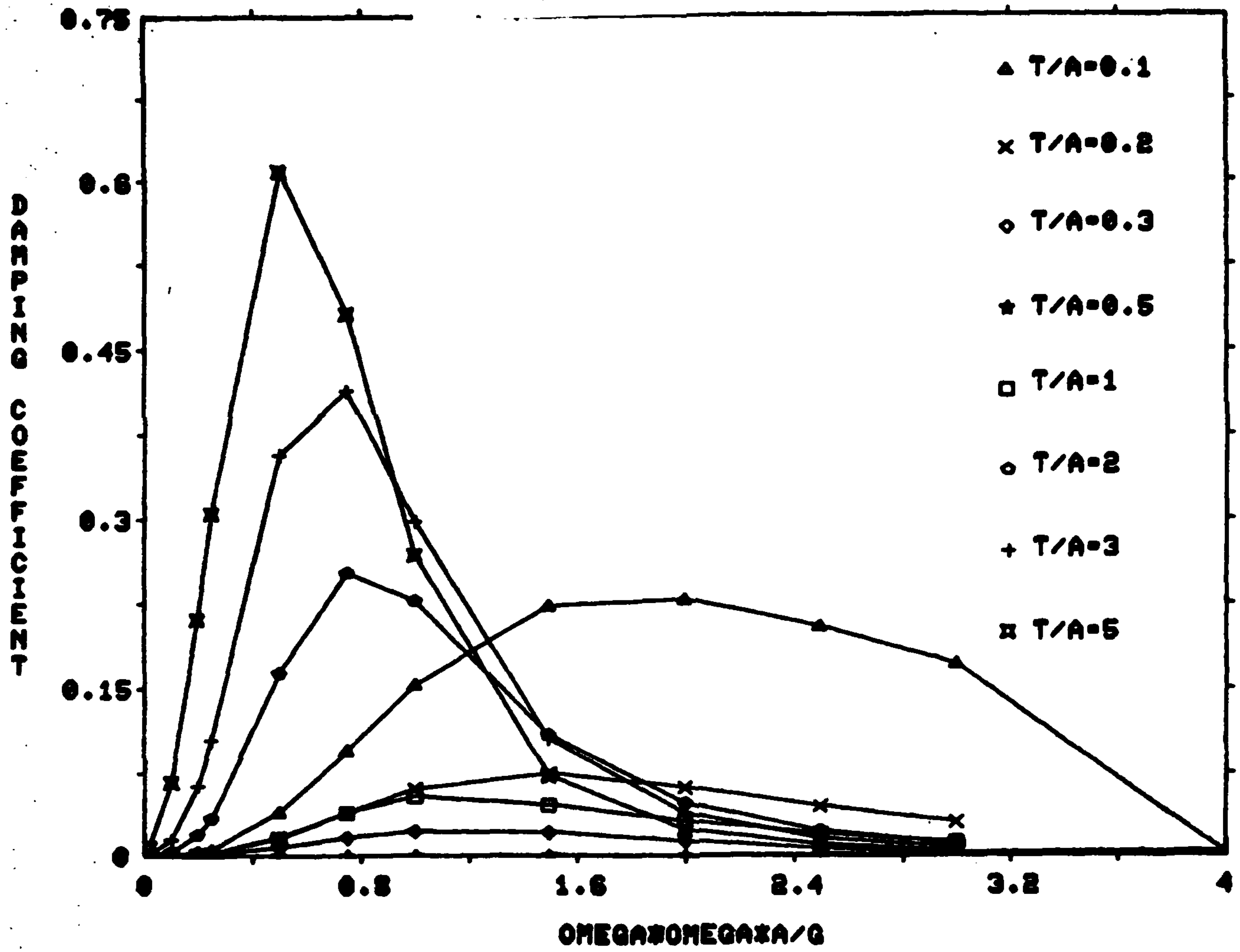


Figure 42

VERTICAL CYLINDER PITCH H/A=20

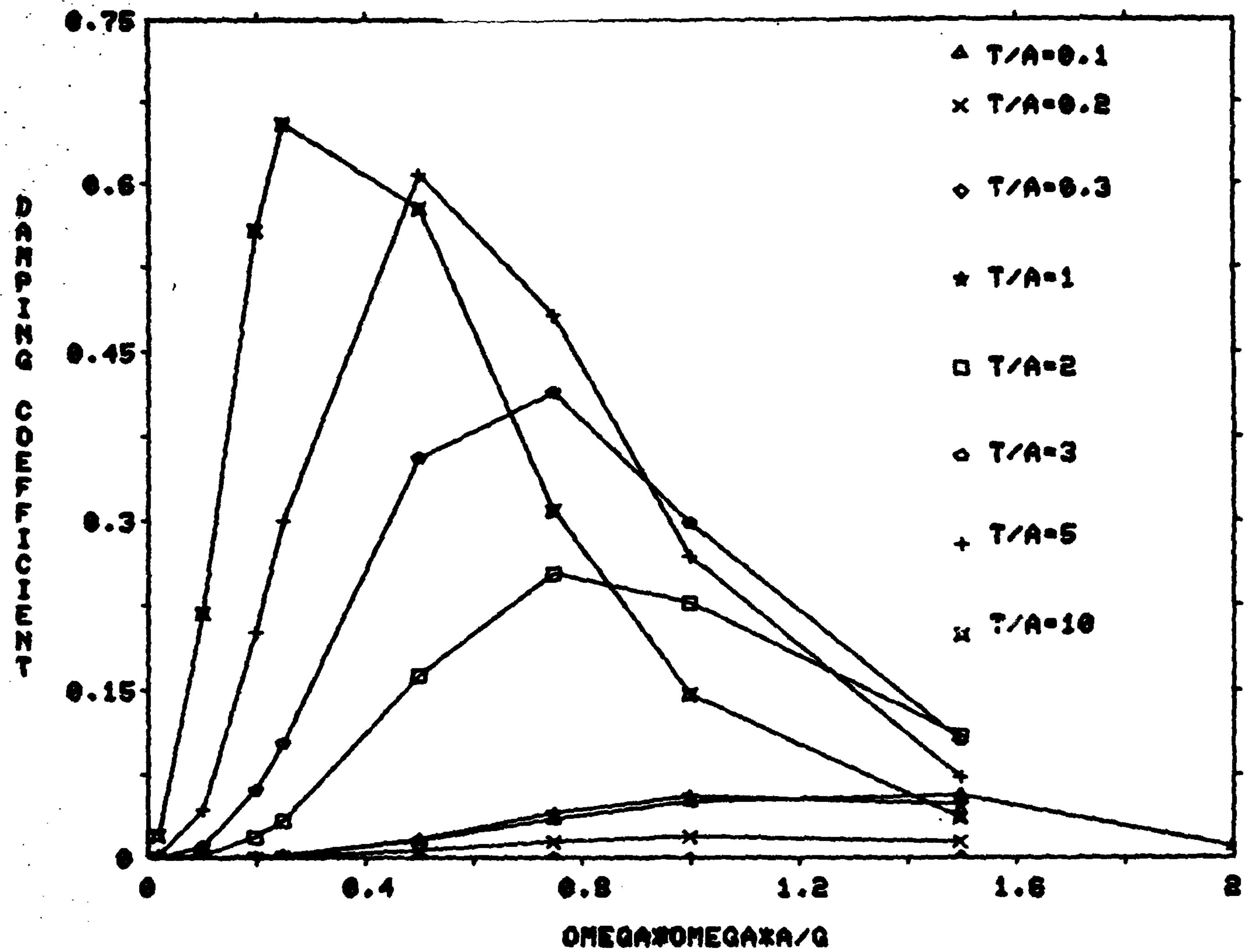


Figure 43

VERTICAL CYLINDER PITCH INDUCED SWAY H/A=3

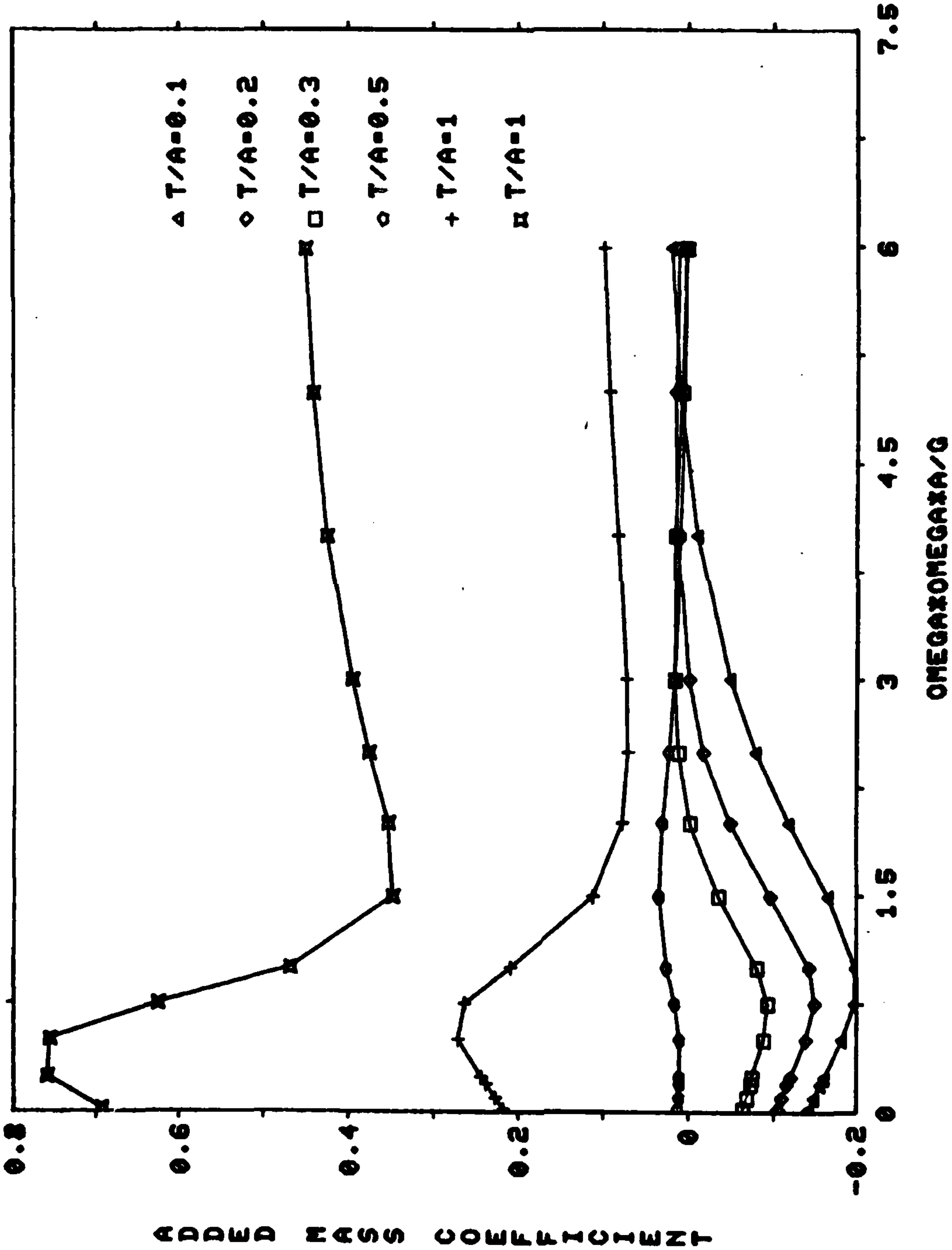


Figure 44

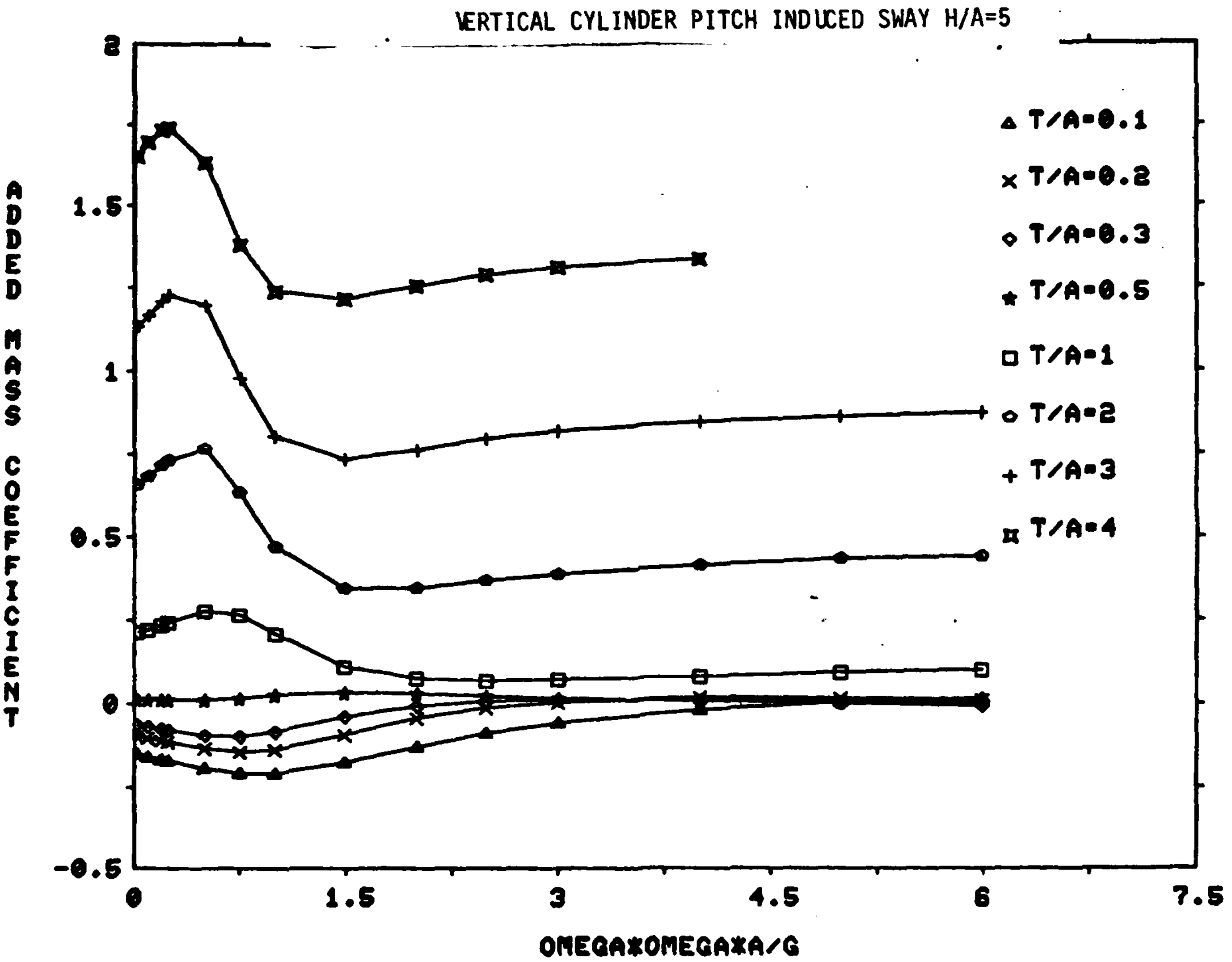

VERTICAL CYLINDER PITCH INDUCED SWAY H/A=5
ADDED MASS COEFFICIENT
2
1.5
1
0.5
0
-0.5
0
1.5
3
4.5
6
7.5
OMEGA*OMEGA*A/G
T/A=0.1
T/A=0.2
T/A=0.3
T/A=0.5
T/A=1
T/A=2
T/A=3
T/A=4

Figure 45

VERTICAL CYLINDER PITCH INDUCED SWAY H/A=5

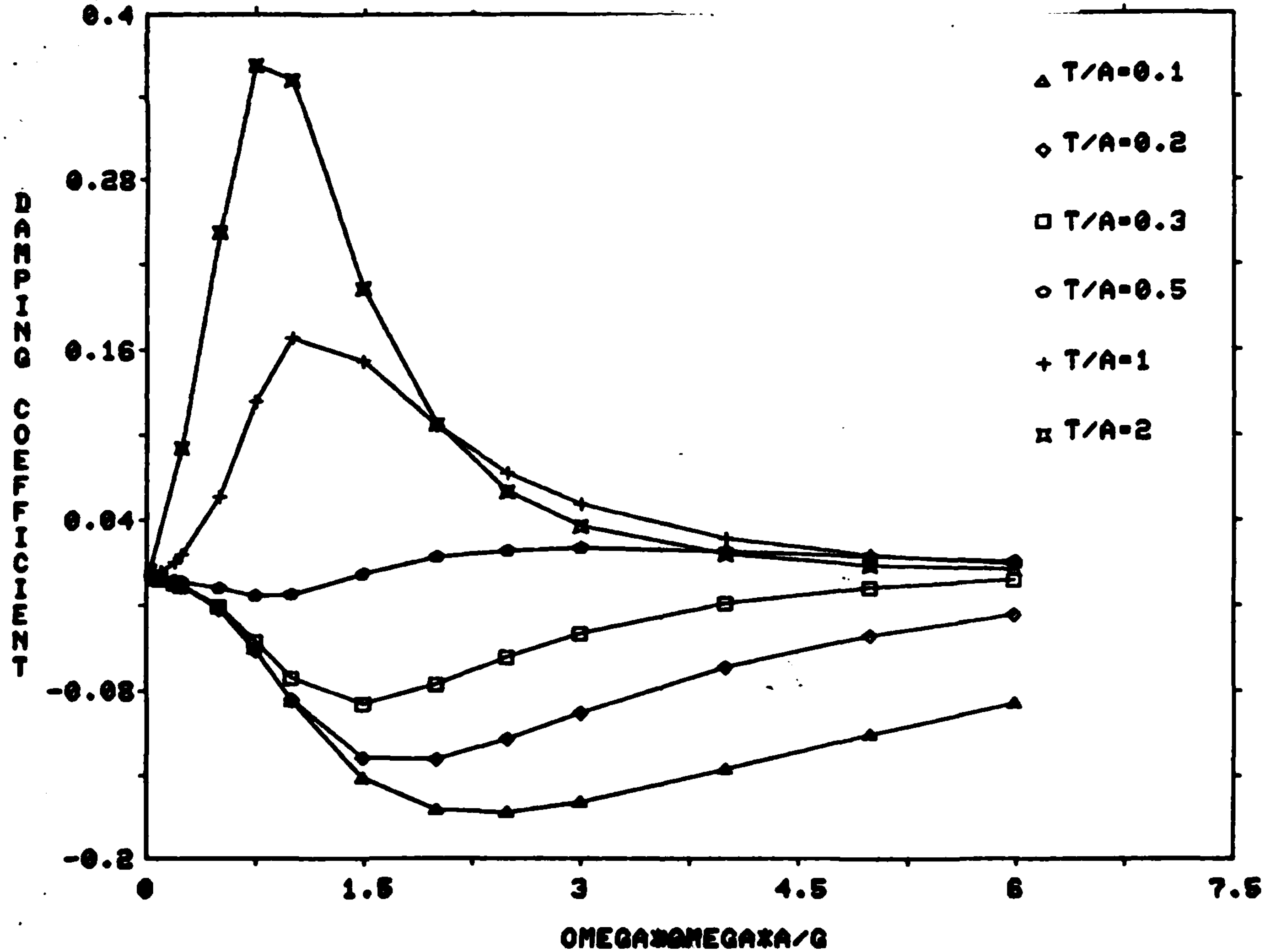


Figure 46

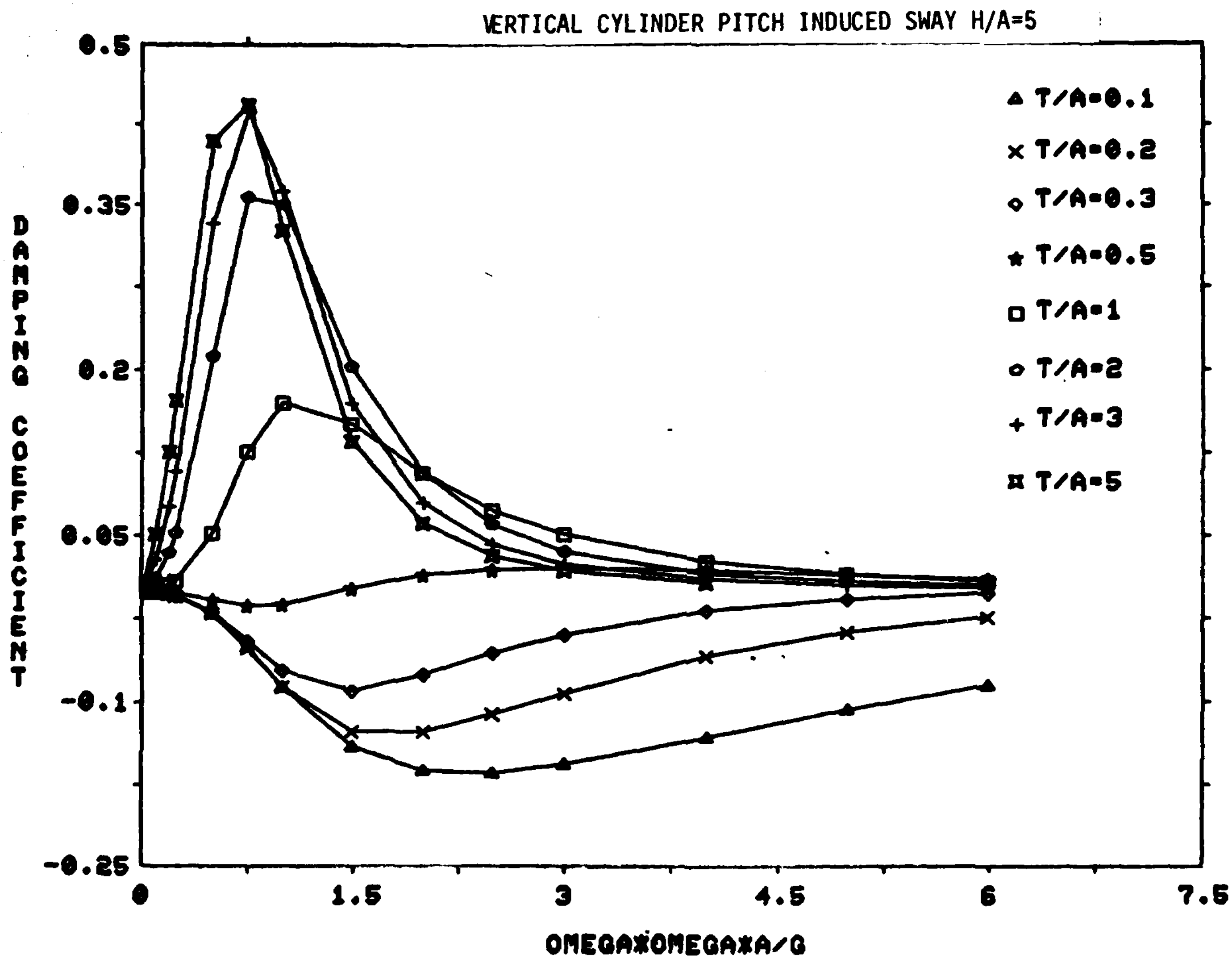

VERTICAL CYLINDER PITCH INDUCED SWAY H/A=5
T/A=0.1
T/A=0.2
T/A=0.3
T/A=0.5
T/A=1
T/A=2
T/A=3
T/A=5
0.5
0.35
0.2
0.05
-0.1
-0.25
0
1.5
3
4.5
6
7.5
OMEGA*OMEGA*A/G
DAMPING COEFFICIENT

Figure 47

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